(Structured) Coding for Real-Time Streaming Communication

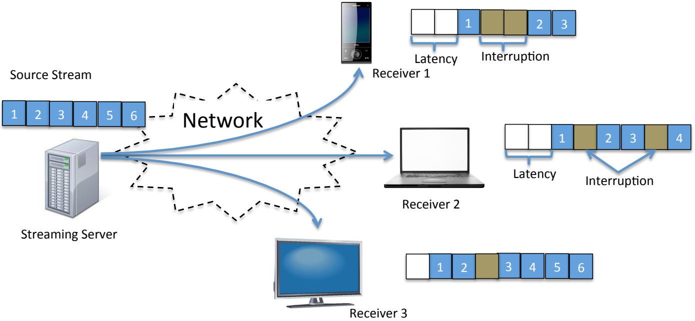
Ashish Khisti

Department of Electrical and Computer Engineering
University of Toronto

Joint work with:
Ahmed Badr (Toronto), Farrokh Etezadi (Toronto), Wai-Tian Tan (Cisco),
John Apostolopoulos (Cisco), Mitchell Trott (Luminate Wireless)

October 9, 2014
University of Michigan, Ann Arbor

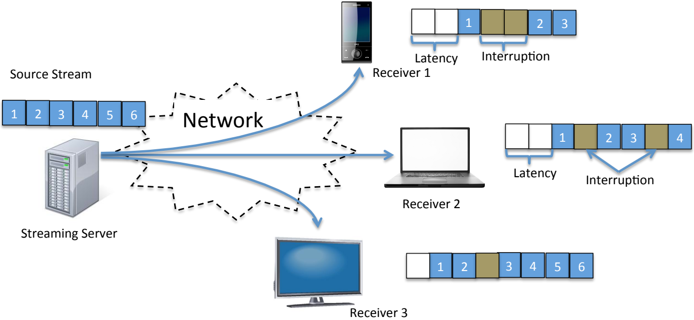
Multimedia Streaming



Applications

Application	Bit-Rate (Mbps)	MSDU (B)	Delay (ms)	PLR
Video Conf.	2 Mbps	1500	100 ms	10^{-4}
Interactive Gaming	1 Mbps	512	50 ms	10^{-4}
Video Streaming	4 Mbps	1500	500 ms	10^{-6}

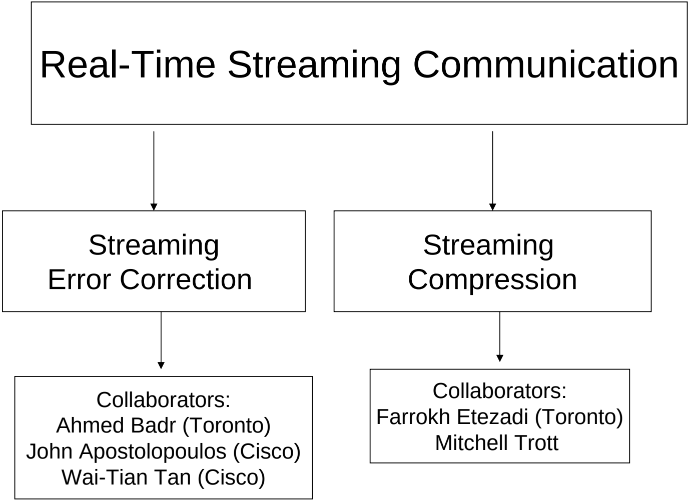
Multimedia Streaming



Packet-Loss Analysis

- Burst Losses - High Performance Degradation
- FEC vs Retransmission

Outline


Real-Time Streaming Communication
Streaming Error Correction
Streaming Compression
Collaborators: Ahmed Badr (Toronto) John Apostolopoulos (Cisco) Wai-Tian Tan (Cisco)
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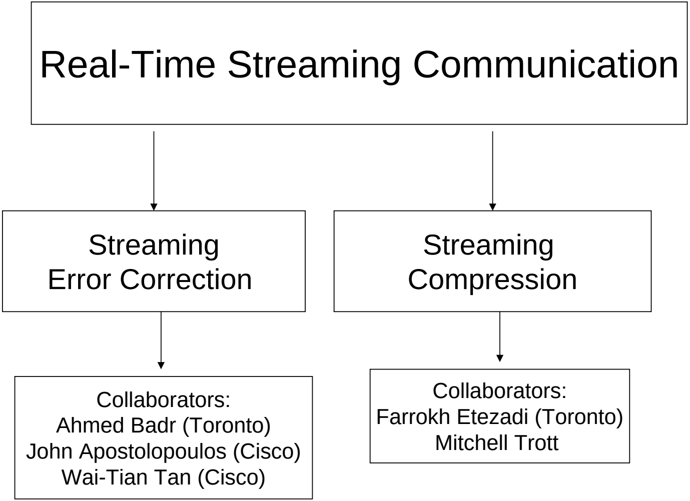
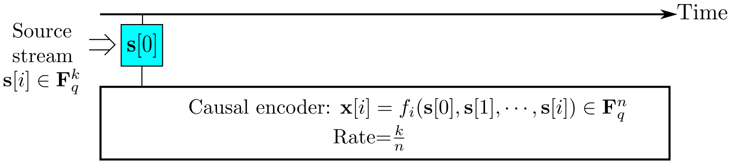


Physical Layer Security (Time Permitting)

Real-Time Communication System

Real-Time Communication System



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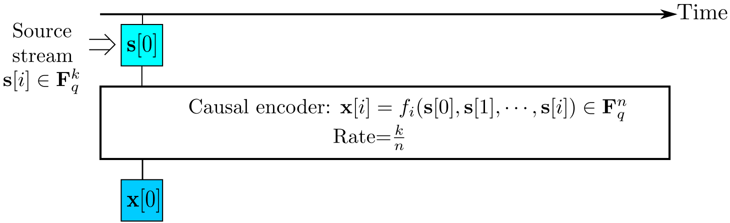

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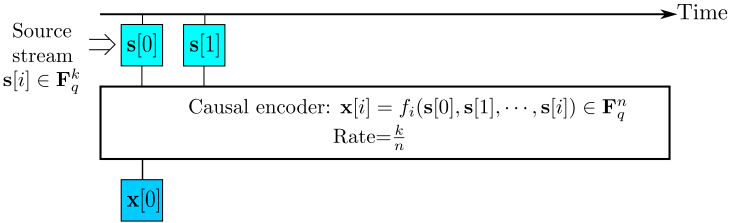

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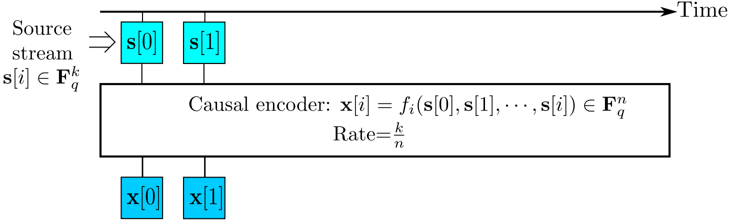

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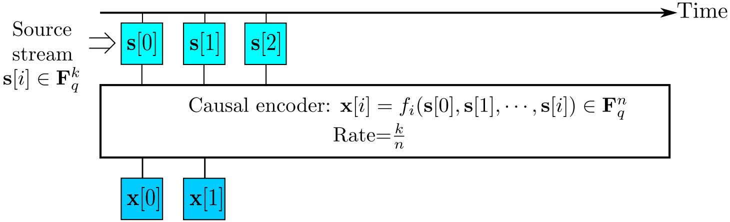

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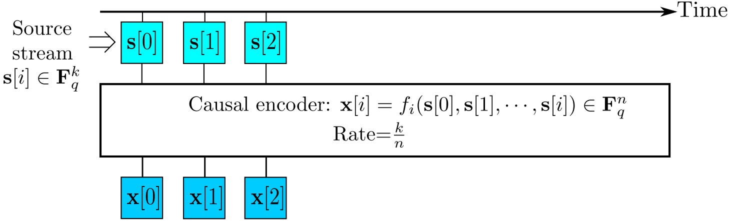

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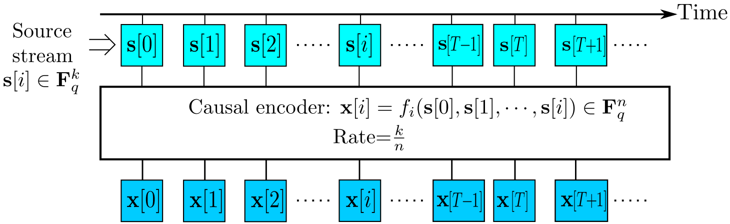

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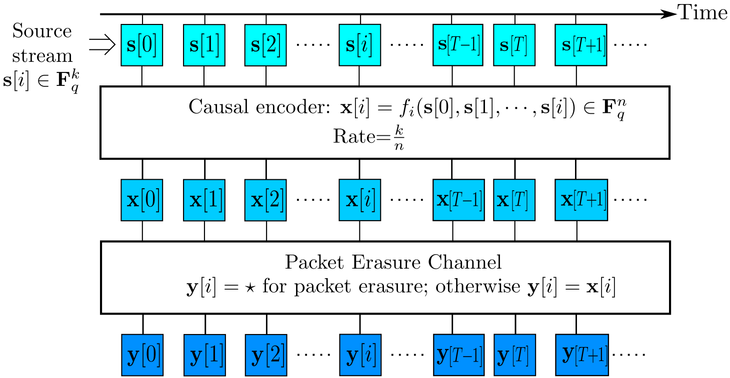

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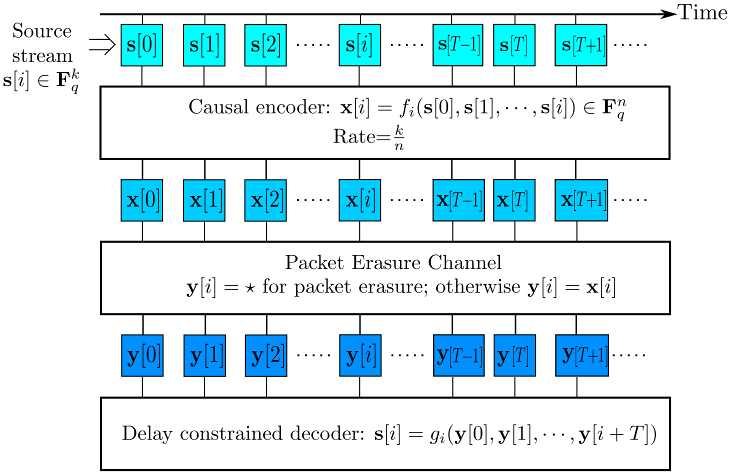

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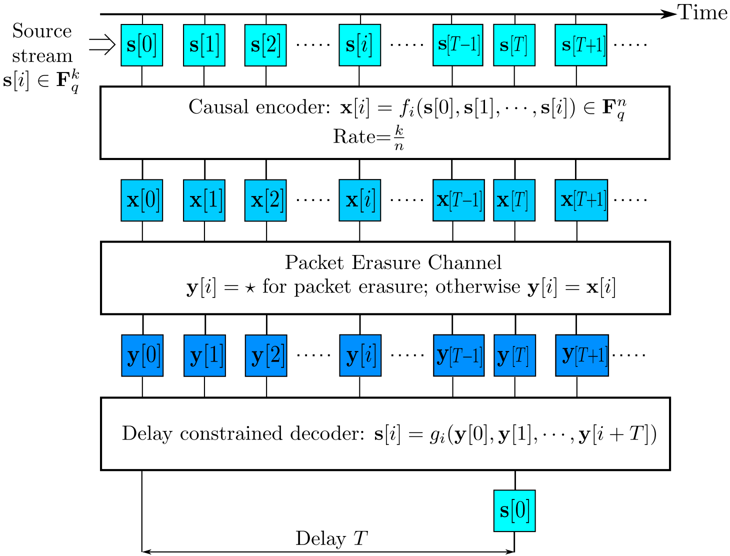

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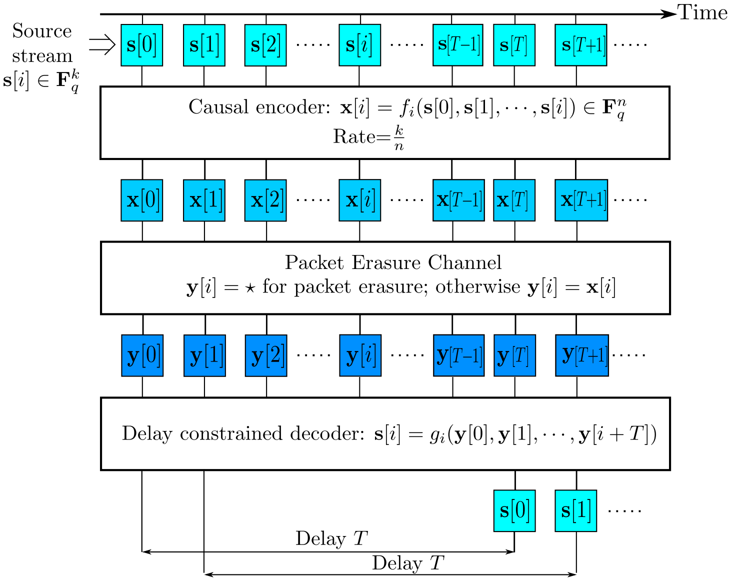
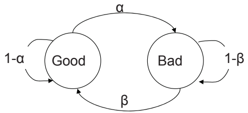


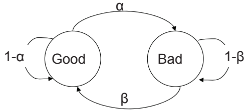
Proposed Channel Model



Gilbert-Elliott Model

- Structural Properties
- Performance Gains

Proposed Channel Model



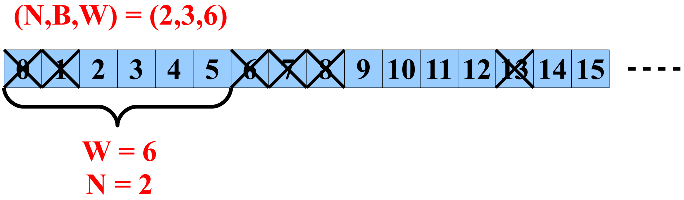
Gilbert-Elliott Model

Sliding Window Erasure Channel: $\mathcal{C}(N, B, W)$

In any sliding window of length W, the channel can introduce only one of the following:

- An erasure burst of maximum length B
- Upto N erasures in arbitrary positions

Sliding Window Erasure Channel: Remarks



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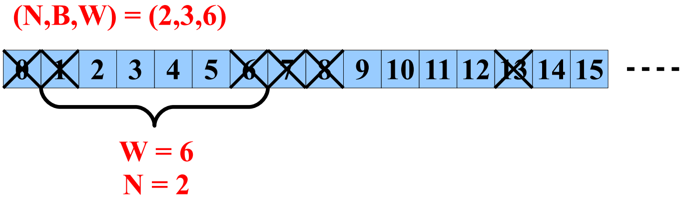

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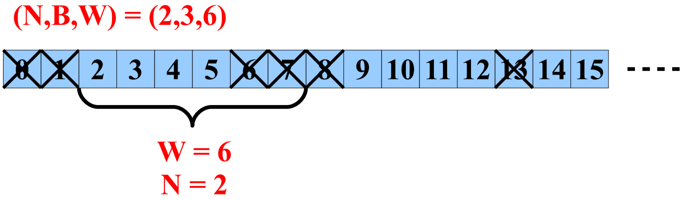

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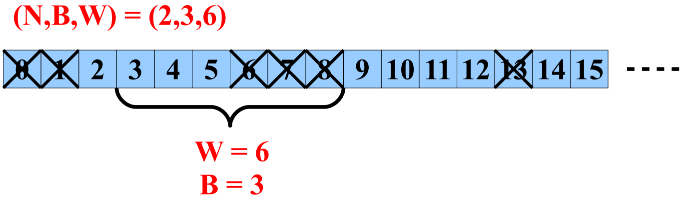

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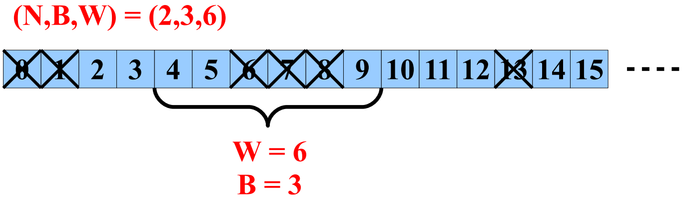

- $\mathcal{C}(N = 1, B, W)$: Burst-Erasure Channel

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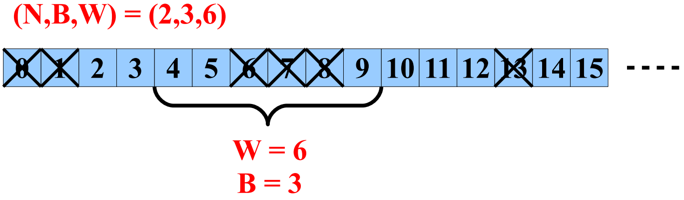

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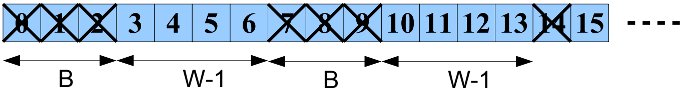
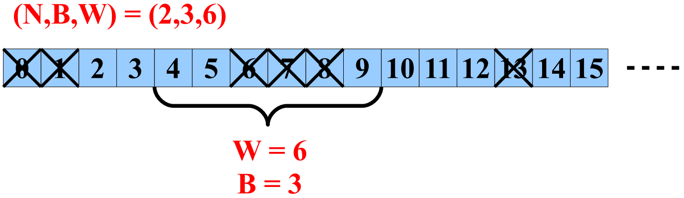
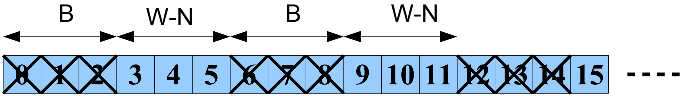




- $\mathcal{C}(N = 1, B, W)$: Burst-Erasure Channel
- $\mathcal{C}(N, B, W)$: "Worst-Case"



Problem Setup - Sliding Window Erasure Channel Model

- Source Model : i.i.d. sequence $\mathsf{s}[t] \sim p_{\mathsf{s}}(\cdot) = \text{Unif}\{(\mathbb{F}_q)^k\}$
- Streaming Encoder: $\mathsf{x}[t] = f_t\left(\mathsf{s}[0], \ldots, \mathsf{s}[t]\right)$, $\mathsf{x}[t] \in (\mathbb{F}_q)^n$
- Erasure Channel: (Sliding Window Model)
- Delay-Constrained Decoder: $\hat{\mathsf{s}}[t] = g_t(\mathsf{y}[0], \ldots, \mathsf{y}[t+T])$
- Rate $R = \frac{k}{n}$

Streaming Capacity

A rate R is achievable over the $\mathcal{C}(N, B, W)$ channel, if there is a sequence of encoding and decoding functions, $f_t(\cdot)$ and $g_t(\cdot)$ respectively over a sufficiently large field $\mathbb{F}_q$, with delay T and rate $R = \frac{k}{n}$. The supremum of achievable rates is the streaming capacity.

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- Worst Case Definition
- Arbitrarily large field size

Main Result

Badr-Patil-Khisti-Tan-Apostolopoulos, IEEE Trans. on Inform. Theory (Under Review)

Theorem

Consider the $\mathcal{C}(N, B, W)$ *channel, with* $W \geq B + 1$*, and let the delay be* T*.*

Upper-Bound *For any rate* R *code, we have:*

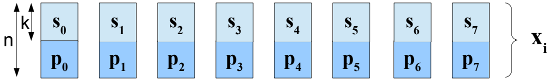
$$\left(\frac{R}{1-R}\right) B + N \leq \min(W, T+1)$$

Lower-Bound: *There exists a rate* R *code that satisfies:*

$$\left(\frac{R}{1-R}\right) B + N \geq \min(W, T+1) - 1.$$

The gap between the upper and lower bound is 1 *unit of delay.*

Baseline Codes - Full Rank Condition



$$\mathbf{p}_i = \mathbf{s}_i \cdot \mathbf{H}_0 + \mathbf{s}_{i-1} \cdot \mathbf{H}_1 + \ldots + \mathbf{s}_{i-M} \cdot \mathbf{H}_M, \qquad \mathbf{H}_i \in \mathbb{F}_q^{k \times n-k}$$

Erasure Codes:

- Random Linear Codes
- Strongly-MDS Codes (Gabidulin'88, Gluesing-Luerssen'06)

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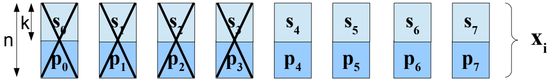

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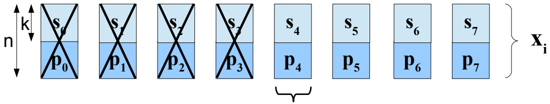

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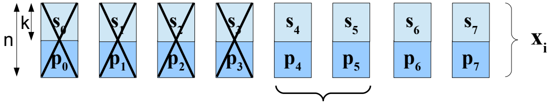

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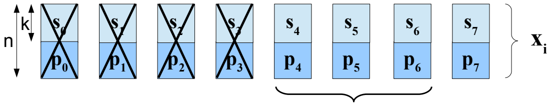

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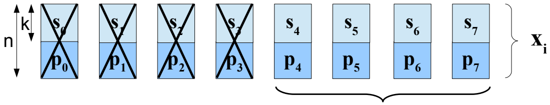

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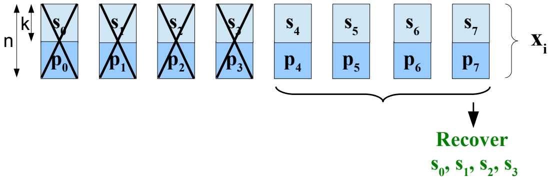

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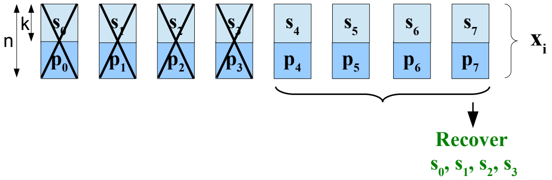

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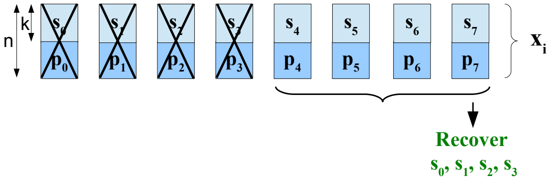

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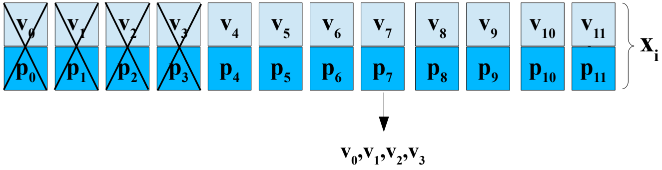
$$\begin{bmatrix} \mathbf{p}_4 \\ \mathbf{p}_5 \\ \mathbf{p}_6 \\ \mathbf{p}_7 \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{H}_4 & \mathbf{H}_3 & \mathbf{H}_2 & \mathbf{H}_1 \\ \mathbf{H}_5 & \mathbf{H}_4 & \mathbf{H}_3 & \mathbf{H}_2 \\ 0 & \mathbf{H}_5 & \mathbf{H}_4 & \mathbf{H}_3 \\ 0 & 0 & \mathbf{H}_5 & \mathbf{H}_4 \end{bmatrix}}_{\text{full rank}} \begin{bmatrix} \mathbf{s}_0 \\ \mathbf{s}_1 \\ \mathbf{s}_2 \\ \mathbf{s}_3 \end{bmatrix}$$

Streaming Code - Example

$B = 4$, $T = 8$, $R = \frac{T}{T+B} = \frac{2}{3}$

Rate $1/2$ Baseline Erasure Codes, $T = 7$



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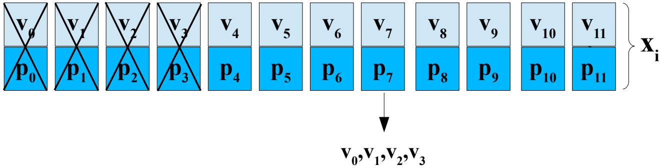
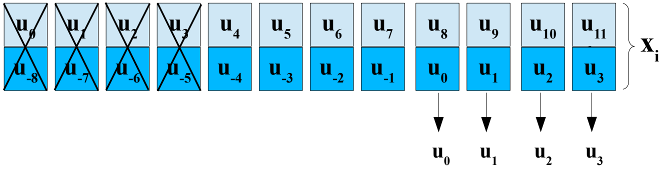


Rate $1/2$ Repetition Code, $T = 8$



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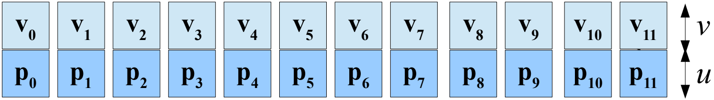
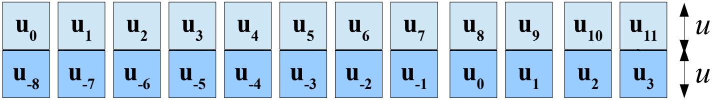




Streaming Code - Example

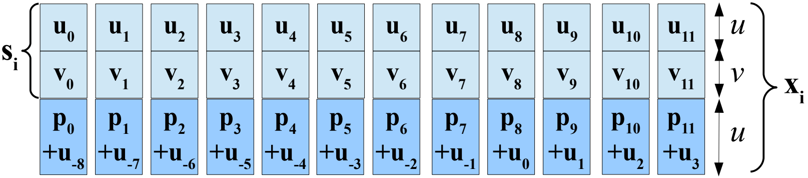
$B = 4$, $T = 8$, $R = \frac{T}{T+B} = \frac{2}{3}$

u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	u
v_0	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}	v_{11}	v
p_0	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	p_{11}	u
u_{-8}	u_{-7}	u_{-6}	u_{-5}	u_{-4}	u_{-3}	u_{-2}	u_{-1}	u_0	u_1	u_2	u_3	u

$$R = \frac{u+v}{3u+v} = \frac{1}{2}$$

Streaming Code - Example

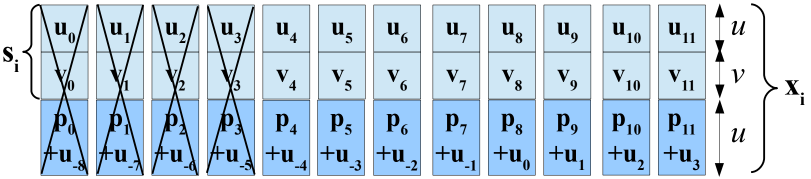
$B = 4$, $T = 8$, $R = \frac{T}{T+B} = \frac{2}{3}$



$$R = \frac{u+v}{2u+v} = \frac{2}{3}$$

Streaming Code - Example

$B = 4$, $T = 8$, $R = \frac{T}{T+B} = \frac{2}{3}$



$$R = \frac{u+v}{2u+v}$$

Encoding:

1. **Source Splitting**: $\mathbf{s}_i = (\mathbf{u}_i, \mathbf{v}_i)$, $\mathbf{u}_i \in \mathbb{F}_q^B$, $\mathbf{v}_i \in \mathbb{F}_q^{T-B}$
2. **Erasure Code on $\mathbf{v}_i$**: Generate $\mathbf{v}_i \rightarrow (\mathbf{v}_i, \mathbf{p}_i)$ where $\mathbf{p}_i \in \mathbb{F}_q^B$ is obtained from a Strongly-MDS code.
3. **Repetition Code on $\mathbf{u}_i$**: Repeat the $\mathbf{u}_i$ symbols with a shift of T
4. **Merging**: Combine the repeated $\mathbf{u}_i$'s with the $\mathbf{p}_i$'s
5. **Rate**: $R = \frac{T}{T+B}$

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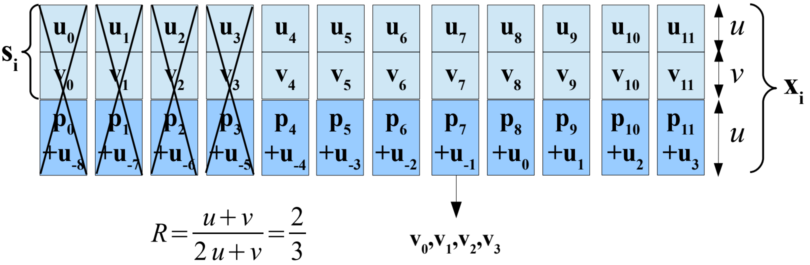
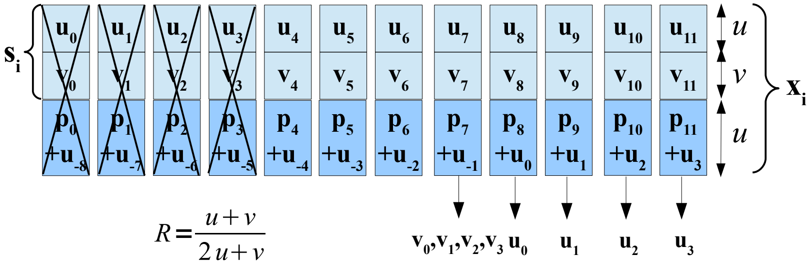


Encoding:

1. **Source Splitting**: $\mathbf{s}_i = (\mathbf{u}_i, \mathbf{v}_i)$, $\mathbf{u}_i \in \mathbb{F}_q^B$, $\mathbf{v}_i \in \mathbb{F}_q^{T-B}$
2. **Erasure Code on $\mathbf{v}_i$**: Generate $\mathbf{v}_i \to (\mathbf{v}_i, \mathbf{p}_i)$ where $\mathbf{p}_i \in \mathbb{F}_q^B$ is obtained from a Strongly-MDS code.
3. **Repetition Code on $\mathbf{u}_i$**: Repeat the $\mathbf{u}_i$ symbols with a shift of T
4. **Merging**: Combine the repeated $\mathbf{u}_i$'s with the $\mathbf{p}_i$'s
5. **Rate**: $R = \frac{T}{T+B}$

Streaming Code - Example

$B = 4$, $T = 8$, $R = \frac{T}{T+B} = \frac{2}{3}$



Encoding:

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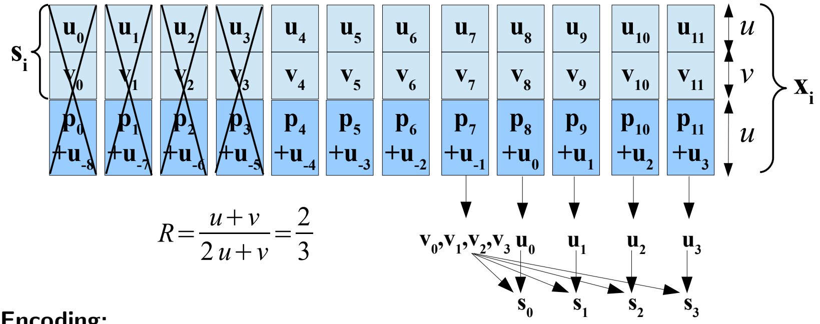
Streaming Code - Example

$B = 4$, $T = 8$, $R = \frac{T}{T+B} = \frac{2}{3}$



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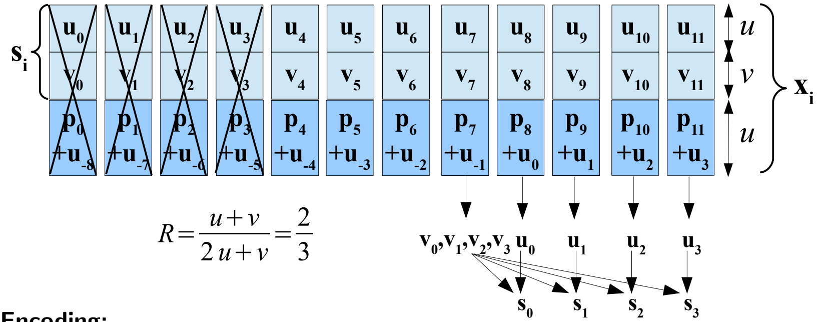
5. **Rate**: $R = \frac{T}{T+B}$

Streaming Code - Example

$B = 4$, $T = 8$, $R = \frac{T}{T+B} = \frac{2}{3}$



$$R = \frac{u+v}{2u+v} = \frac{2}{3}$$

Encoding:

1. **Source Splitting**: $\mathbf{s}_i = (\mathbf{u}_i, \mathbf{v}_i)$, $\mathbf{u}_i \in \mathbb{F}_q^B$, $\mathbf{v}_i \in \mathbb{F}_q^{T-B}$
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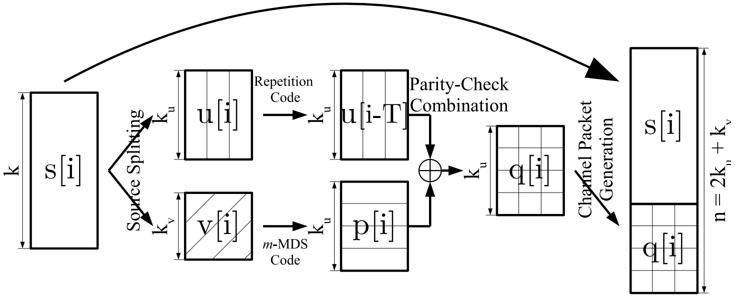

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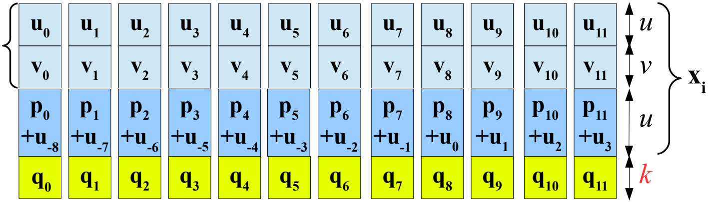
Robust Extension: $\mathcal{C}(N, B, W)$ Channel

Layered Code Design



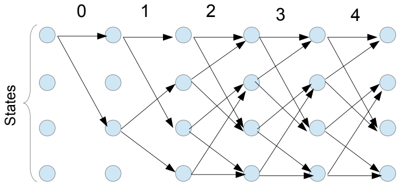
- **Burst-Erasure Streaming Code**: $(\mathbf{u}_i, \mathbf{v}_i, \mathbf{p}_i + \mathbf{u}_{i-T})$
- **Erasure Code**: $\mathbf{q}_i = \sum_{t=1}^{M} \mathbf{u}_{i-t} \cdot \mathbf{H}_t^u, \qquad \mathbf{q}_i \in \mathbb{F}_q^k$
- **Concatenation**: $(\mathbf{u}_i, \mathbf{v}_i, \mathbf{p}_i + \mathbf{u}_{i-T}, \mathbf{q}_i)$

$$R = \frac{T}{T+B+k}$$

- Attains the lower bound

Distance and Span Properties

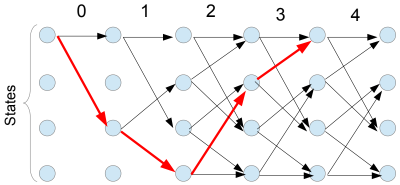
Consider (n, k, m) Convolutional code: $\mathbf{x}_i = \sum_{j=0}^{m} \mathbf{s}_{i-j}\mathbf{G}_j$



Trellis Diagram

Distance and Span Properties

Consider (n, k, m) Convolutional code: $\mathbf{x}_i = \sum_{j=0}^{m} \mathbf{s}_{i-j} \mathbf{G}_j$



Trellis Diagram – Free Distance

Distance and Span Properties

Consider (n, k, m) Convolutional code: $\mathbf{x}_i = \sum_{j=0}^{m} \mathbf{s}_{i-j}\mathbf{G}_j$

Column Distance: d_T

$$d_T = \min_{\substack{[\mathbf{s}_0,\ldots,\mathbf{s}_T] \\ \mathbf{s}_0 \neq 0}} \text{wt}\left(\begin{bmatrix} \mathbf{s}_0 & \ldots & \mathbf{s}_T \end{bmatrix} \begin{bmatrix} \mathbf{G}_0 & \mathbf{G}_1 & \ldots & \mathbf{G}_T \\ 0 & \mathbf{G}_0 & \ldots & \mathbf{G}_{T-1} \\ \vdots & & \ddots & \vdots \\ 0 & & \ldots & \mathbf{G}_0 \end{bmatrix} \right)$$

Distance and Span Properties

Consider (n, k, m) Convolutional code: $\mathbf{x}_i = \sum_{j=0}^{m} \mathbf{s}_{i-j} \mathbf{G}_j$

Column Distance: d_T

$$d_T = \min_{\substack{[\mathbf{s}_0, \ldots, \mathbf{s}_T] \\ \mathbf{s}_0 \neq 0}} \mathrm{wt} \left(\begin{bmatrix} \mathbf{s}_0 & \ldots & \mathbf{s}_T \end{bmatrix} \begin{bmatrix} \mathbf{G}_0 & \mathbf{G}_1 & \ldots & \mathbf{G}_T \\ 0 & \mathbf{G}_0 & \ldots & \mathbf{G}_{T-1} \\ \vdots & & \ddots & \vdots \\ 0 & & \ldots & \mathbf{G}_0 \end{bmatrix} \right)$$

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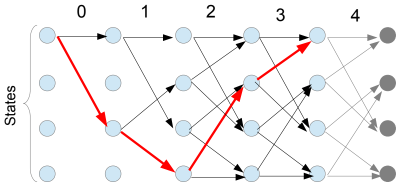

Column Span in [0,3]

Distance and Span Properties

Consider (n, k, m) Convolutional code: $\mathbf{x}_i = \sum_{j=0}^{m} \mathbf{s}_{i-j}\mathbf{G}_j$

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$$d_T = \min_{\substack{[\mathbf{s}_0,\ldots,\mathbf{s}_T] \\ \mathbf{s}_0 \neq 0}} \mathrm{wt}\left(\begin{bmatrix} \mathbf{s}_0 & \ldots & \mathbf{s}_T \end{bmatrix} \begin{bmatrix} \mathbf{G}_0 & \mathbf{G}_1 & \ldots & \mathbf{G}_T \\ 0 & \mathbf{G}_0 & \ldots & \mathbf{G}_{T-1} \\ \vdots & & \ddots & \vdots \\ 0 & & \ldots & \mathbf{G}_0 \end{bmatrix} \right)$$

Column Span: c_T

$$c_T = \min_{\substack{[\mathbf{s}_0,\ldots,\mathbf{s}_T] \\ \mathbf{s}_0 \neq 0}} \mathrm{span}\left(\begin{bmatrix} \mathbf{s}_0 & \ldots & \mathbf{s}_T \end{bmatrix} \begin{bmatrix} \mathbf{G}_0 & \mathbf{G}_1 & \ldots & \mathbf{G}_T \\ 0 & \mathbf{G}_0 & \ldots & \mathbf{G}_{T-1} \\ \vdots & & \ddots & \vdots \\ 0 & & \ldots & \mathbf{G}_0 \end{bmatrix} \right)$$

Column-Distance & Column Span Tradeoff

Badr-Patil-Khisti-Tan-Apostolopoulos '2014

Theorem

Consider a $\mathcal{C}(N, B, W)$ channel with delay T and $W \geq T+1$. A streaming code is feasible over this channel if and only if it satisfies: $d_T \geq N+1$ and $c_T \geq B+1$

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Theorem

For any rate R convolutional code and any $T \geq 0$ the Column-Distance d_T and Column-Span c_T satisfy the following:

$$\left(\frac{R}{1-R}\right) c_T + d_T \leq T + 1 + \frac{1}{1-R}$$

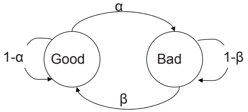
There exists a rate R code (MiDAS Code) over a sufficiently large field that satisfies:

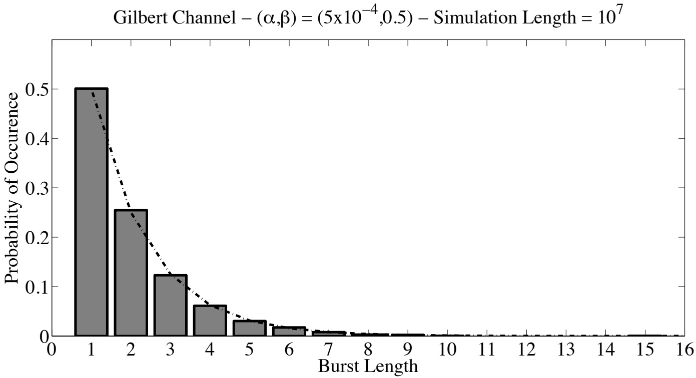
$$\left(\frac{R}{1-R}\right) c_T + d_T \geq T + \frac{1}{1-R}$$

Gilbert-Elliott Channel $(\alpha, \beta) = (5 \times 10^{-4}, 0.5)$, $T = 12$ and $R \approx 0.5$

Gilbert Elliott Channel

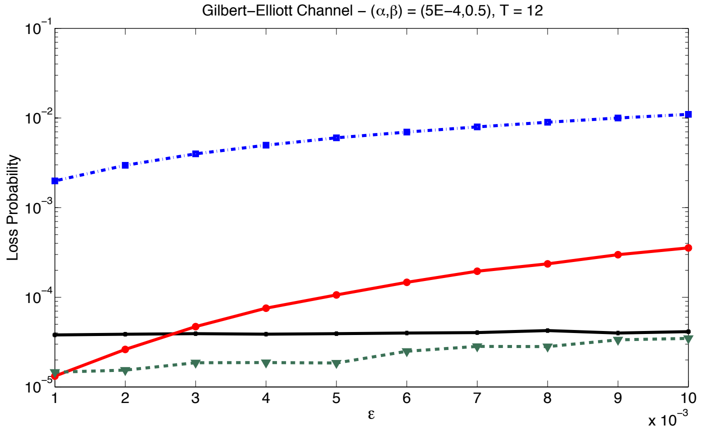
- Good State: $\Pr(\text{loss}) = \varepsilon$
- Bad State: $\Pr(\text{loss}) = 1$





Simulation Results

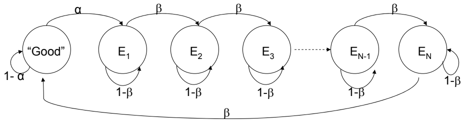
Gilbert-Elliott Channel $(\alpha, \beta) = (5 \times 10^{-4}, 0.5)$, $T = 12$ and $R \approx 0.5$



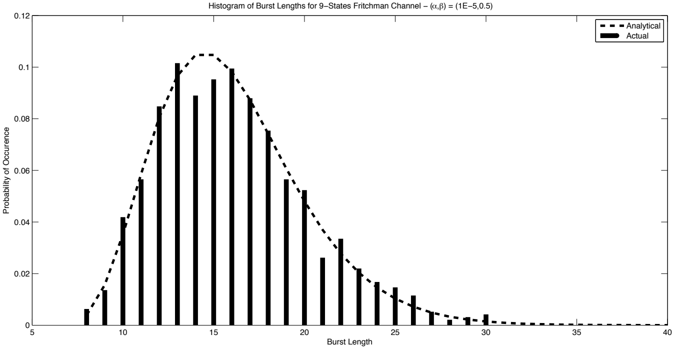
Code	N	B	Code	N	B
Strongly MDS	6	6	MiDAS	2	9
Burst-Erasure	1	11			

Simulation Results-II

Fritchman Channel $(\alpha, \beta) = (1e-5, 0.5)$ and $T = 40$ and $R = 40/79$, 9 states



- $\alpha = 1e-5$
- $\beta = 0.5$



Simulation Results-II

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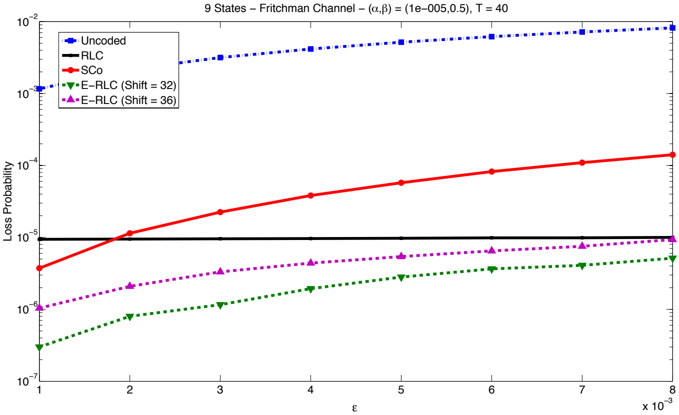
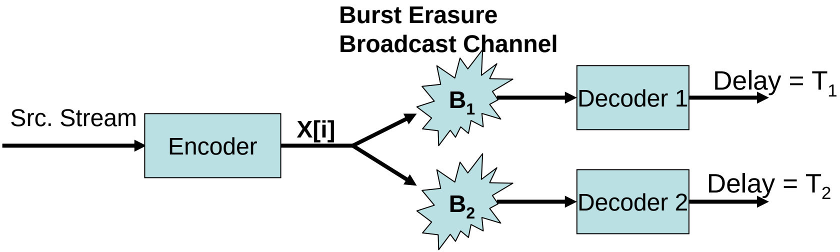


Code	N	B	Code	N	B
Strongly MDS	20	20	MiDAS-I	8	31
Burst Erasure	1	39	MiDAS-II	4	35

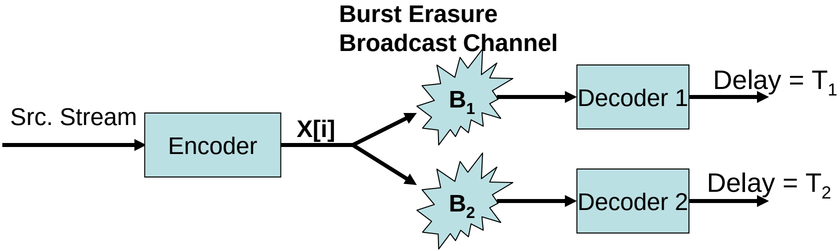
Multicast Streaming Codes



Motivation

- $B_1 < B_2$
- Receiver 1 : Good Channel State
- Receiver 2: Weaker Channel State
- Delay adapts to Channel State

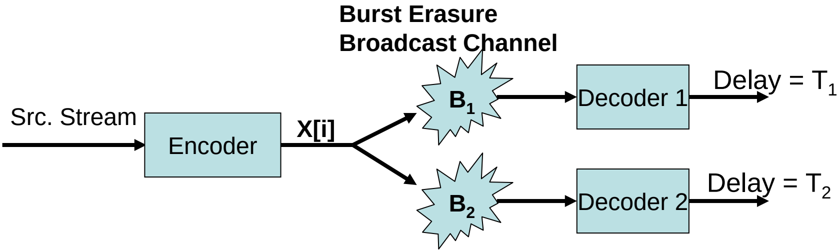
Multicast Streaming Codes



Capacity Function

- Capacity function $C(T_1, T_2, B_1, B_2)$
- Single User Upper Bound: $C \leq \min\left(\frac{T_1}{T_1+B_1}, \frac{T_2}{T_2+B_2}\right)$
- Concatenation Lower Bound: $C \geq \frac{1}{1+\frac{B_1}{T_1}+\frac{B_2}{T_2}}$

Multicast Streaming Setup



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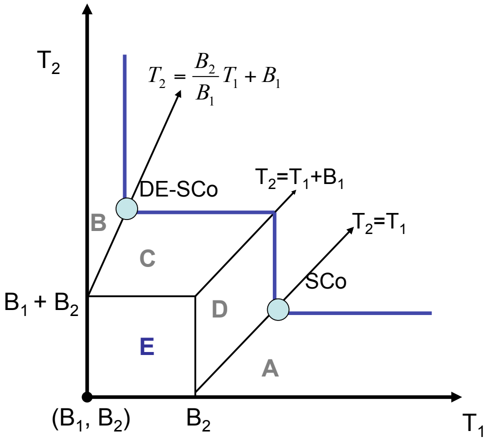
Multicast Streaming Capacity

Badr-Khisti-Lui (IT Trans. To Appear 2014)

Assume w.l.o.g. $B_2 \geq B_1$



Region	Capacity
A	$\frac{T_2}{T_2 + B_2}$
B	$\frac{T_1}{T_1 + B_1}$
C	$\frac{T_2 - B_1}{T_2 - B_1 + B_2}$
D	$\frac{T_1}{T_1 + B_2}$
E	Partial Characterization

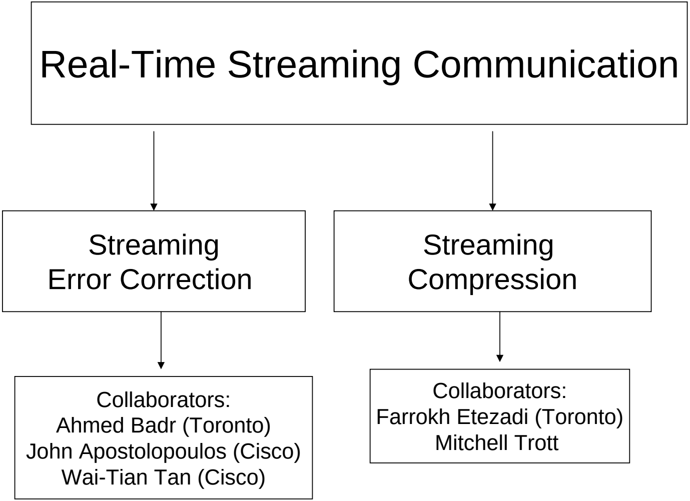
Other Extensions

- Mismatched Streaming Codes (Patil-Badr-Khisti-Tan Asilomar 2013)
- Partial Recovery Streaming Codes (Badr-Khisti-Tan-Apostolopoulos JSTSP 2014)
- Multiple Erasure Bursts (Li-Khisti-Girod Asilomar 2011) - Interleaved Low-Delay Codes
- Multiple Links (Lui-Badr-Khisti CWIT 2011) - Layered coding for burst erasure channels
- Multiple Source Streams with Different Decoding Delays (Lui (Unpublished) 2011) - Embedded Codes

Other Results

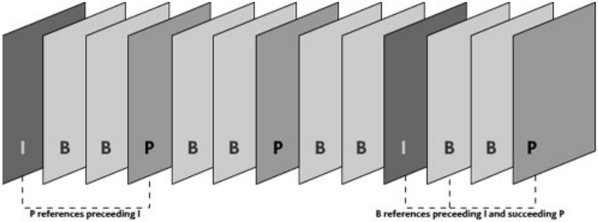
- Burst Erasure Channels: Martinian and Sundberg (IT-2004)
- Other Recent Results: Leong-Ho (ISIT 2012), Leong-Qureshi-Ho (ISIT 2013)

Outline


Real-Time Streaming Communication
Streaming
Error Correction
Streaming
Compression
Collaborators:
Ahmed Badr (Toronto)
John Apostolopoulos (Cisco)
Wai-Tian Tan (Cisco)
Collaborators:
Farrokh Etezadi (Toronto)
Mitchell Trott

Compression Vs Error Propagation

GOP Picture Structure[1]

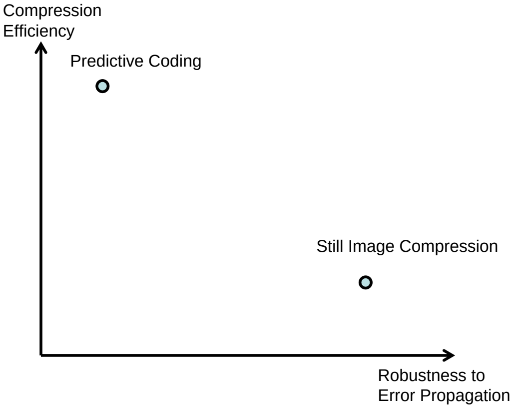


	Compression	Error Propagation
Predictive Coding	$\surd$	$\times$
Still Image Coding	$\times$	$\surd$

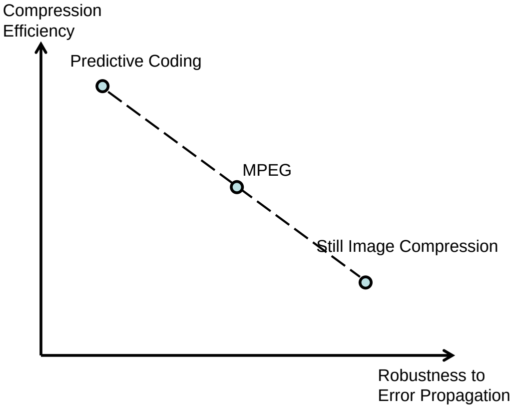
- Interleaving Approach
- Error Control Coding

[1] Source : `http://www.networkwebcams.com`

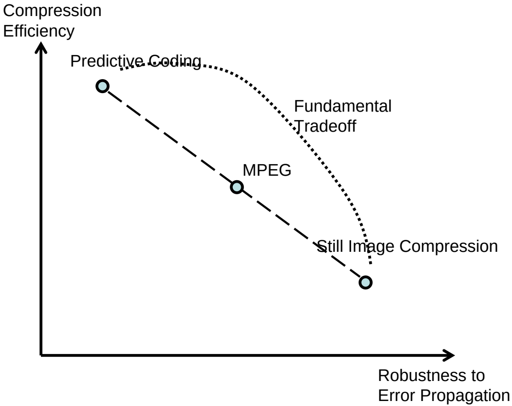
Motivation - Video Streaming



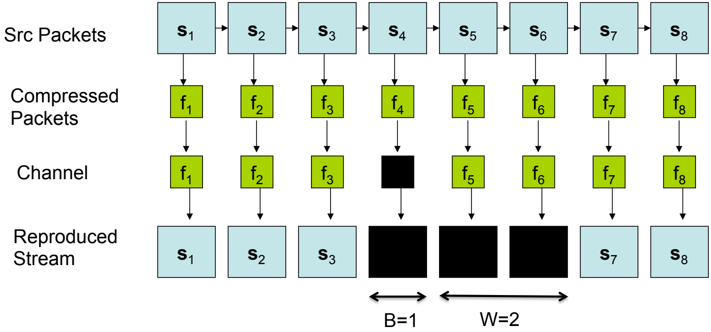
Motivation - Video Streaming



Motivation - Video Streaming



Information Theoretic Model



System Parameters:

- Compression Rate: R
- Erasure Burst Length: B
- Recovery Window: W
- Lossless or Lossy Recovery

Problem Setup

- Source Model: Sequence of vectors — Temporally Markov, Spatially i.i.d. (see Viswanathan and Berger ('00), Wang and Xu ('10), Song-Chen-Wang-Liu ('13), Ma-Ishwar ('11))

$$\Pr(\mathsf{s}_i^n | \mathsf{s}_{i-1}^n, \mathsf{s}_{i-2}^n, \ldots,) = \prod_{j=1}^{n} \Pr(\mathsf{s}_{ij} | \mathsf{s}_{i-1,j})$$

- Channel Model: Burst Erasure Model

$$g_i = \begin{cases} f_i, & i \notin \{j, j+1, \ldots, j+B-1\} \\ \star, & \text{otherwise} \end{cases}$$

Problem Setup

- Source Model: Sequence of vectors — Temporally Markov, Spatially i.i.d. (see Viswanathan and Berger ('00), Wang and Xu ('10), Song-Chen-Wang-Liu ('13), Ma-Ishwar ('11))

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- Channel Model: Burst Erasure Model

$$g_i = \begin{cases} f_i, & i \notin \{j, j+1, \ldots, j+B-1\} \\ \star, & \text{otherwise} \end{cases}$$

- Encoder: $\mathcal{F}_i : \{\mathsf{s}_0^n, \ldots, \mathsf{s}_i^n\} \to f_i \in \{1, 2, \ldots, 2^{nR}\}$.
- Decoder: $\mathcal{G}_i : \{g_0, \ldots, g_i\} \to \hat{\mathsf{s}}_i^n$
  Lossless Recovery:

$$\Pr(\mathsf{s}_i^n \neq \hat{\mathsf{s}}_i^n) \leq \varepsilon_n$$

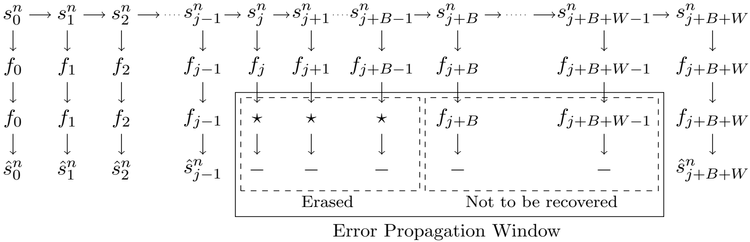
except for $i \in [j, \ldots, j+B+W-1]$.

Rate-Recovery Function

Definition (Rate-Recovery Function)

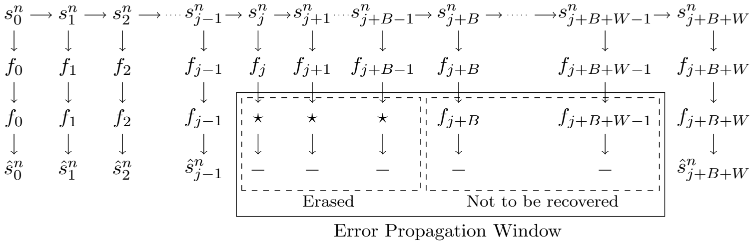
The minimum compression rate R that is achieved when:

- *Burst-Erasure Length $= B$*
- *Recovery Window $= W$*
- *Lossless or Lossy Recovery*



Rate-Recovery Function

Etezadi-Khisti-Trott (IEEE Trans. Information Theory, Aug. 2014)



Theorem (Upper and Lower Bounds - Lossless Case)

$$R^{+}(B,W) = H(\mathsf{s}_1|\mathsf{s}_0) + \frac{1}{W+1} I(\mathsf{s}_B;\mathsf{s}_{B-1}|\mathsf{s}_{-1})$$

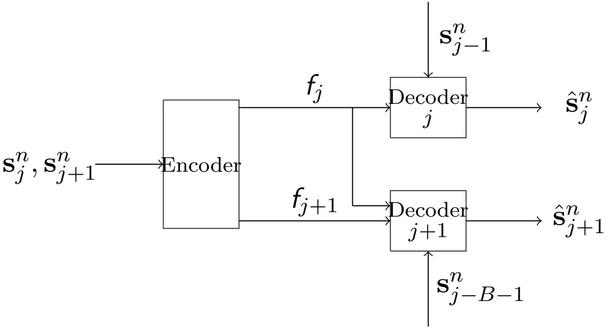
$$R^{-}(B,W) = H(\mathsf{s}_1|\mathsf{s}_0) + \frac{1}{W+1} I(\mathsf{s}_{B+W};\mathsf{s}_{B-1}|\mathsf{s}_{-1})$$

- Upper bound : Binning based scheme.
- Upper and Lower Bounds Coincide: $W = 0$ and $W \to \infty$.

Lower Bound - Rate Recovery Function

Etezadi-Khisti-Trott (IEEE Trans. Information Theory, Aug. 2014)

Let $W = 1$. Encoding of s_j^n, s_{j+1}^n



Lower Bound - Rate Recovery Function

Etezadi-Khisti-Trott (IEEE Trans. Information Theory, Aug. 2014)

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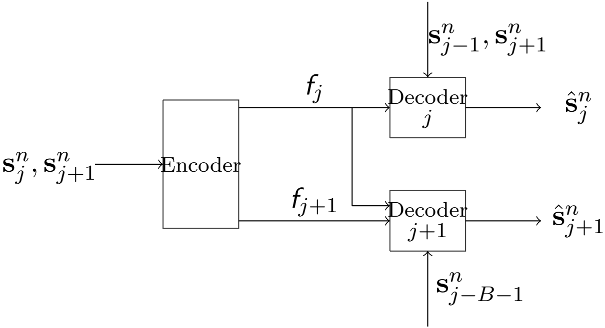
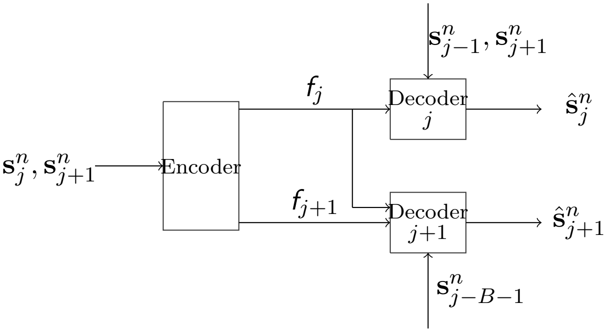


Lower Bound - Rate Recovery Function

Etezadi-Khisti-Trott (IEEE Trans. Information Theory, Aug. 2014)

Let $W = 1$. Encoding of s_j^n, s_{j+1}^n



Lower Bound: $R_j + R_{j+1} \geq H(s_j | s_{j-1}, s_{j+1}) + H(s_{j+1} | s_{j-B-1})$.

Gauss-Markov Source

Etezadi-Khisti-Trott (IEEE Trans. Information Theory, Aug. 2014)

Stationary Gauss-Markov Source, $s_i \sim \mathcal{N}(0, 1)$.

$$s_i = \rho s_{i-1} + z_i, \quad z_i \sim \mathcal{N}(0, 1 - \rho^2) \perp \{s_j\}_{j<i}$$

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Quadratic Distortion Measure:

$$E\left[\frac{1}{n}\sum_{k=1}^{n}\left(s_i(k) - \hat{s}_i(k)\right)^2\right] \leq D,$$

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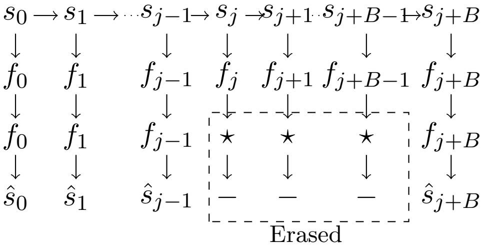

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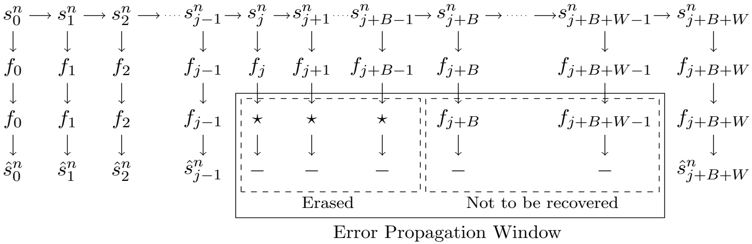
No Recovery Period, $W = 0$

- Lossy Rate Recovery Function $R(B, D)$
- High Resolution Limit $R(B, D) = \frac{1}{2}\log\left(\frac{1-\rho^{2(B+1)}}{D}\right) + o_D(1)$.
- Upper and Lower bounds.
- $W > 0$: Novel Hybrid Coding Schemes

Gauss-Markov Sources
$W > 0$ (In Progress)

- Predictive Coding: $u_i = s_i - \hat{s}_i(u_{-\infty}^{i-1}) + n_i$
- Memoryless Quantize and Binning: $u_i = s_i + n_i$
- Truncated Prediction (Hybrid): $u_i = s_i - \hat{s}_i(u_{i-W}^{i}) + n_i$



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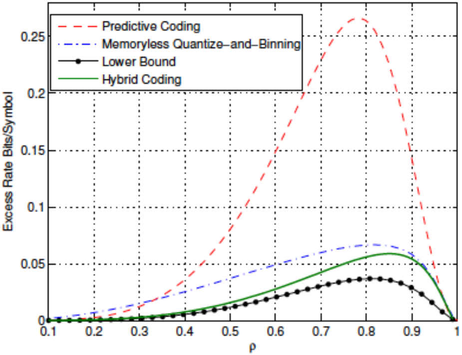
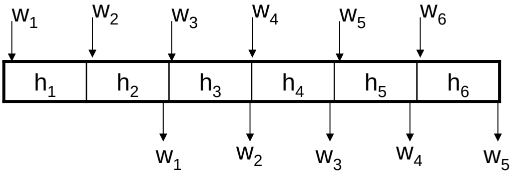


(a) $B = W = 1$

Block Fading Channels



Block Fading Channel

$$\mathbf{y}_i = h_i \mathbf{x}_i + \mathbf{z}_i, \qquad i = 1, 2, \ldots$$

- Block Fading Channels: n symbols per block
- Source Packet: One in each coherence block nR bits
- Decoding Delay: T coherence blocks
- Quasi-Static Model: $T = 1$
- Ergodic Model $T \to \infty$

Streaming DMT of Fading Channels

Theorem (Khisti-Draper , IT-Trans To Appear, 2014)

The diversity multiplexing tradeoff for streaming source with a delay of T coherence blocks and a block-fading channel model is

$$d(r) = Td_1(r)$$

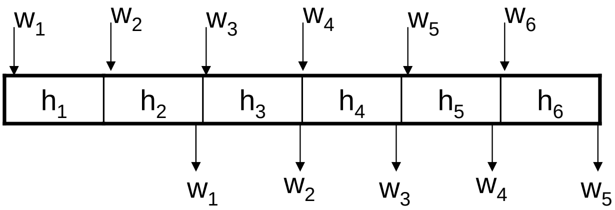
where $d_1(r)$ is the quasi-static DMT.

- Coding Scheme: Time-Sharing
- Upper Bound: Outage Amplification Argument

Achievability: Fading Channels

$T = 2$

- $T-$ parallel channel code
- Divide each coherence block into T sub-blocks
- Apply parallel channel code across T sub-blocks



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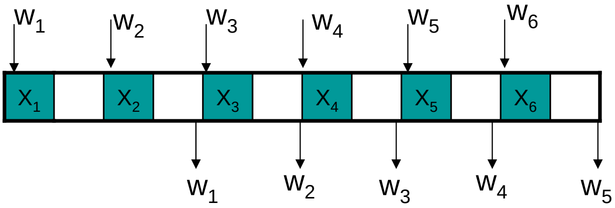

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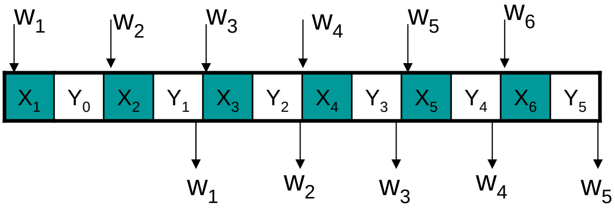

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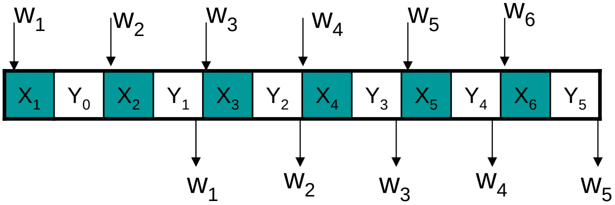
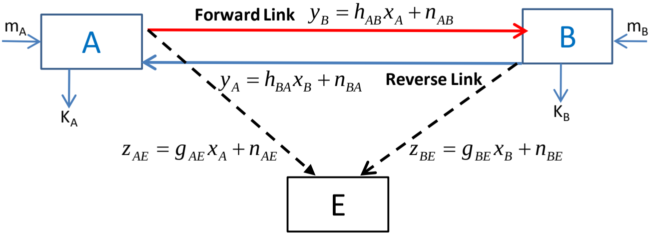


DMT: $d(r) = 2 - 2r$

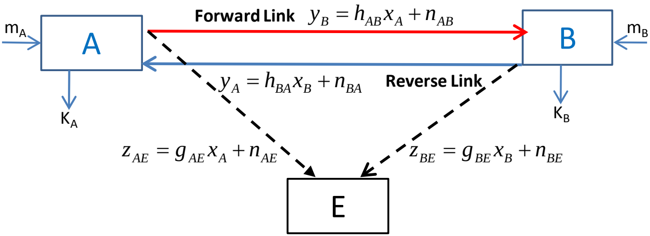
Physical Layer Security



- Two-way block-fading channel
- Coherence Period: T
- Channel Reciprocity
- Non Coherent Main-Channel, Perfect CSI for eavesdropper
- Interactive Communication

Secret-Key Generation over Fading Channels

Khisti, IT-Trans Submitted Aug. 2013



High SNR Capacity:

$$C \approx \frac{1}{T} I(h_{AB}; h_{BA}) + E\left[\log\left(1 + \frac{|h_{AB}|^2)}{|g_A|^2}\right)\right] + E\left[\log\left(1 + \frac{|h_{BA}|^2}{|g_B|^2)}\right)\right]$$

Optimal Scheme: Training + Interactive Communication

Secure Communication - Wiretap Channel

- Secure Multi-Antenna Multicast (A. Khisti IT-2011, A. Khisti and D. Zhang Comm Letters 2013)
- Practical Codes for MIMO Wiretap Channel (A. Khina and Y. Kochman ISIT 2014)
  - V-Blast Type Decomposition
  - Scalar AWGN Wiretap Codebooks
- MIMO Arbitrarily Varying Wiretap Channel (A. Yener and X. He, IT-Trans 2013, T-COMM 2014)
- Fading Wiretap Channel with Imperfect CSI (Z. Rezki and M. S. Alouini, T-COMM 2014, T-Wireless 2014)
- Private Broadcasting (T. Liu IT-Trans 2014)
- Secure Broadcasting with Independent Secret Keys (R. Schaefer, CISS 2014)

Conclusions

Real-Time Streaming Communication

- Part I: Channel Coding
  - Channels with Burst and Isolated Erasures
  - Explicit Codes
  - New Distance and Span Metrics
- Part II: Source Coding
  - Tradeoff between compression rate and error propagation
  - Lossless Recovery
  - Gaussian Sources with Quadratic Distortion