

# Social Footprints: Identifying the roles of nodes and links in massive social networks

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Hayder Radha

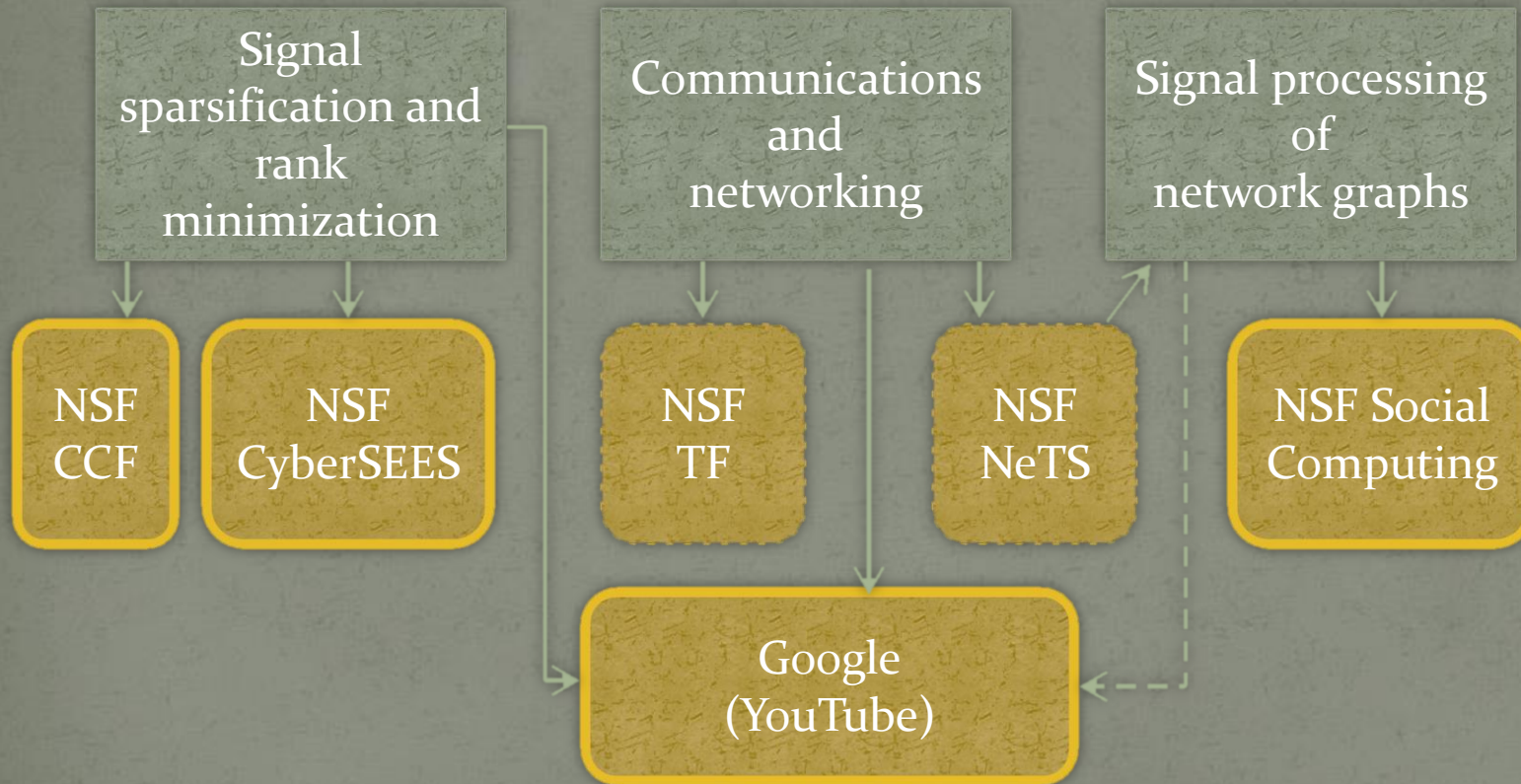
Michigan State University

WAVES Lab

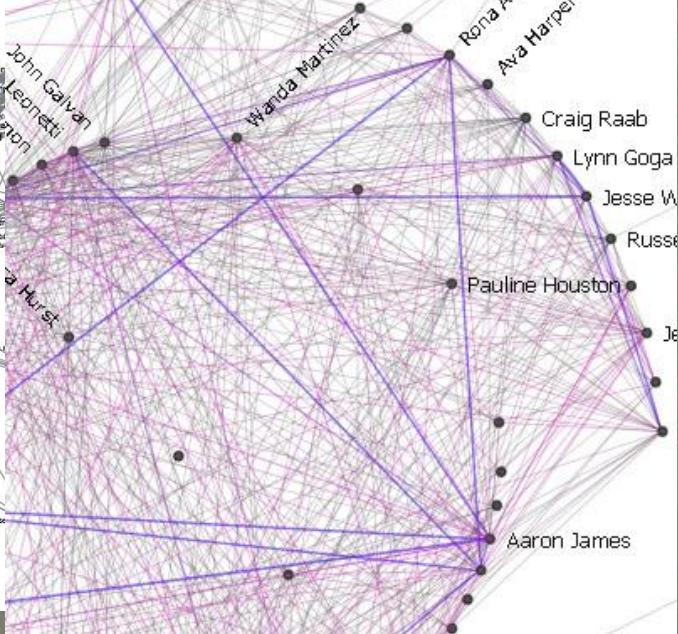
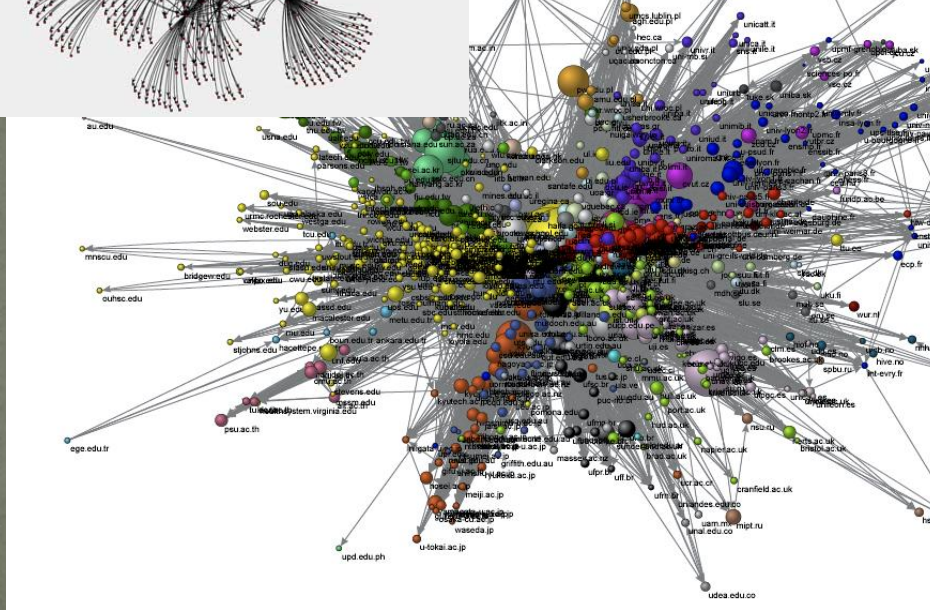
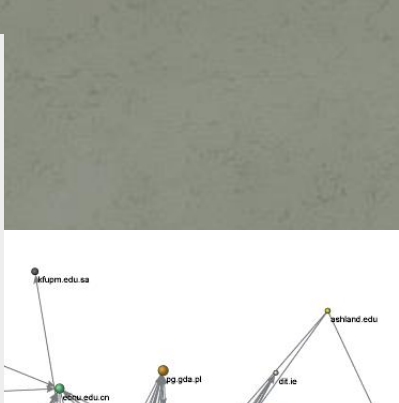
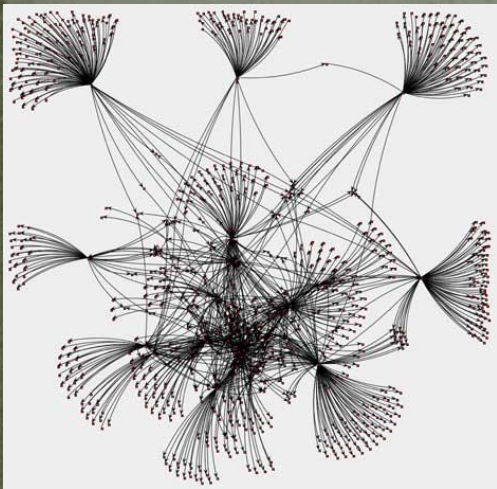
# Outline

- How do we identify the “influence” of nodes in a network?
  - How do we identify the “influence” of nodes from structural information only (topology of a network)
- Can we identify more specific roles of individuals in complex social networks?
  - Influence models between leaders and followers
  - What type of leaders?
- How to measure the “importance” and “role” of links?
  - “Strength of weak links”
  - Closing the gap between macro- and micro- analysis

# WAVES Lab: Research Areas



# Network Graphs

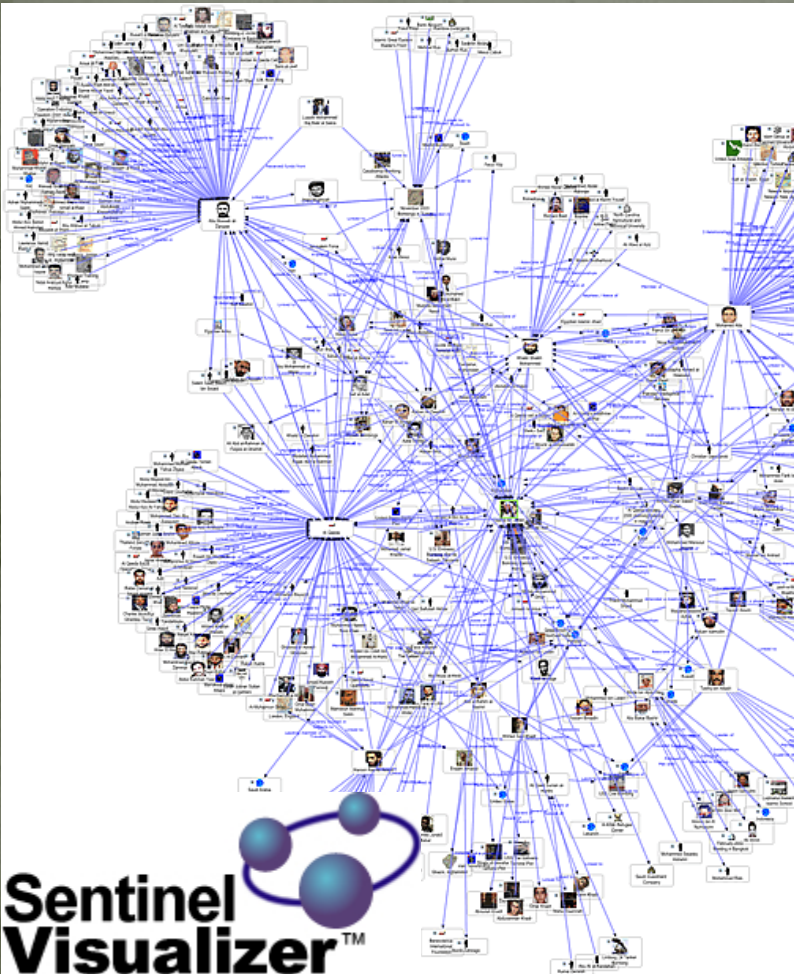


Michigan State

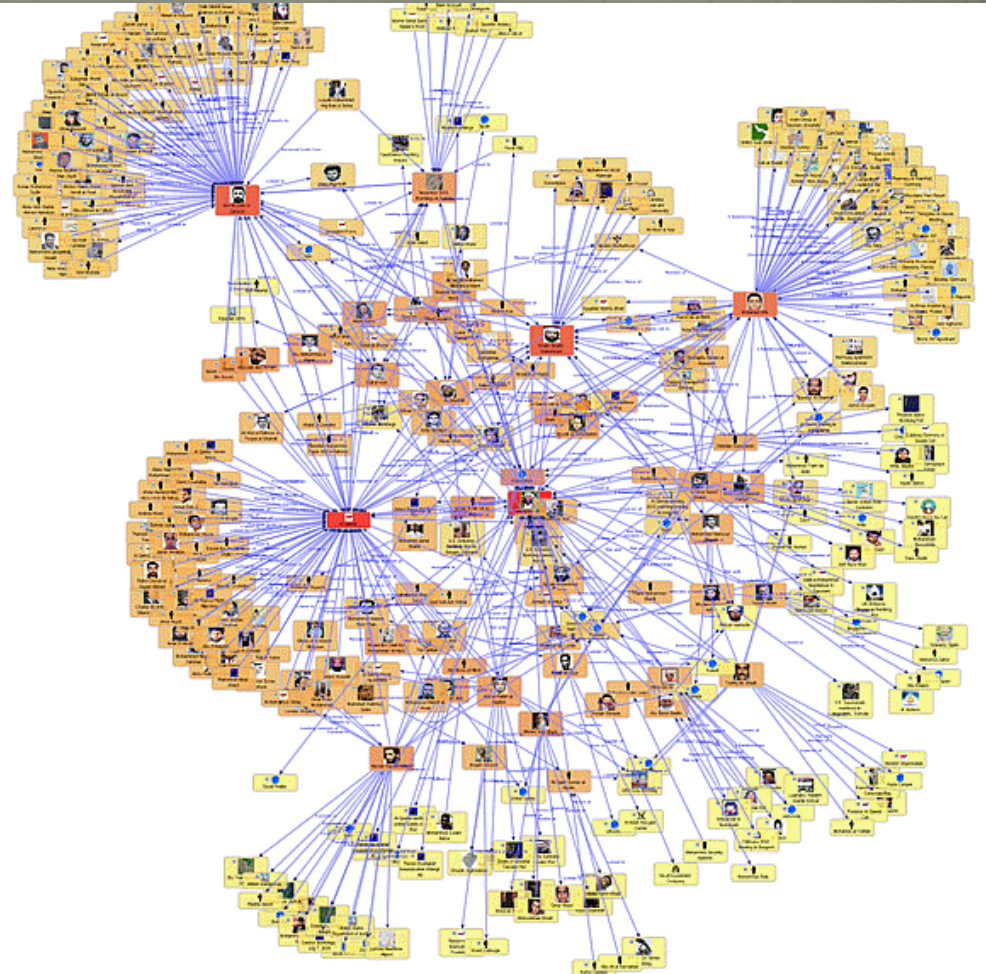
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# Network Graphs



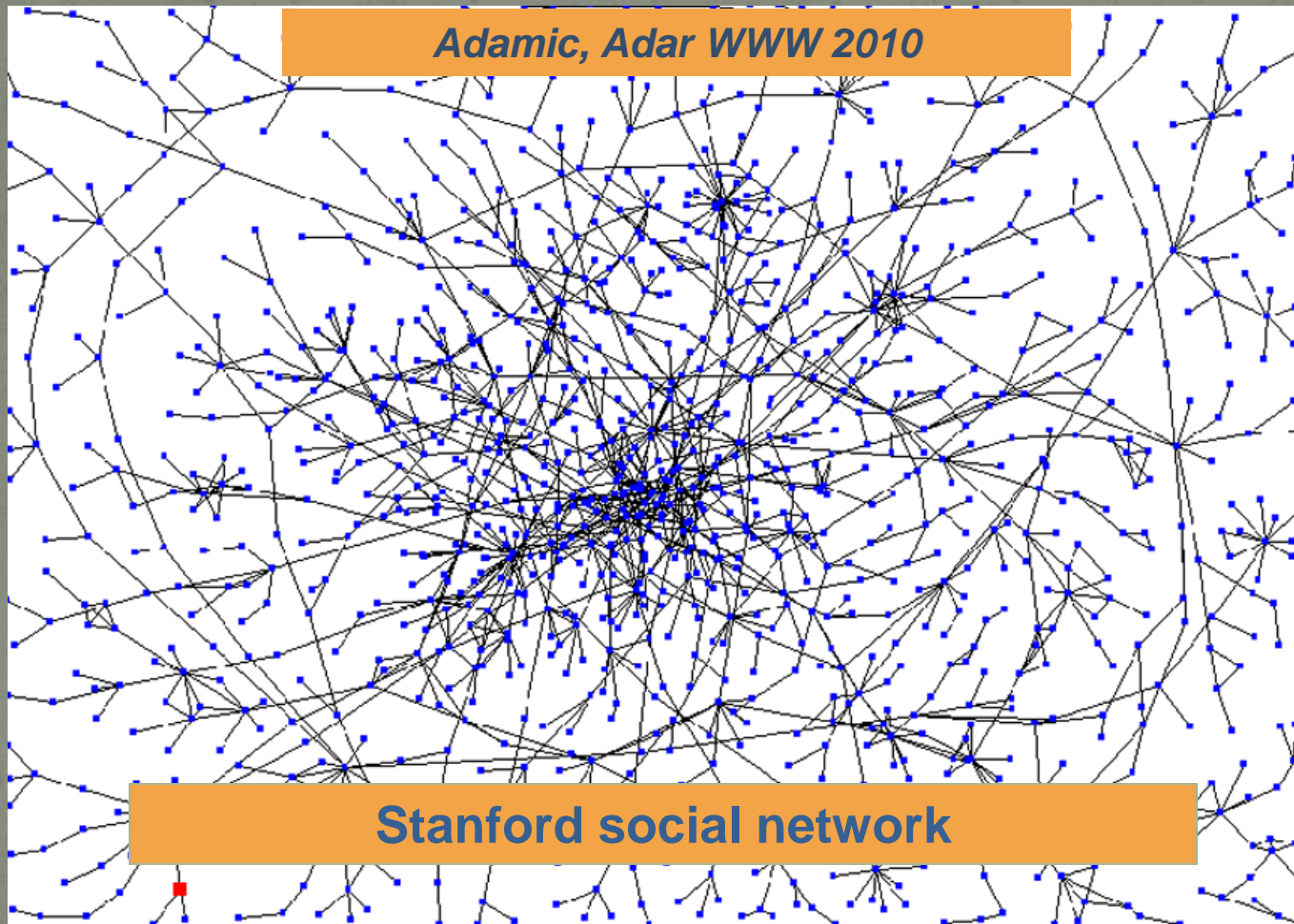
Michigan State



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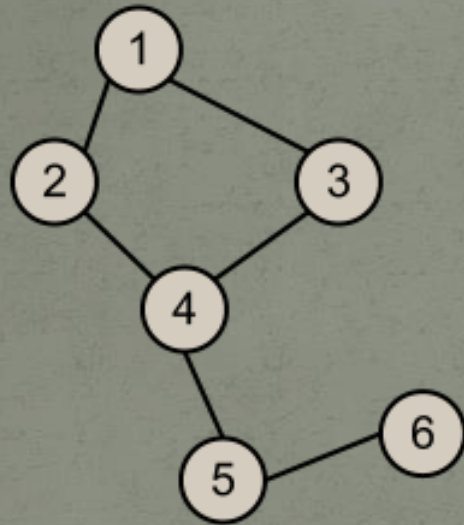
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# Identifying “influential” friends



# Graphs and adjacency matrices

## Undirected Graph & Adjacency Matrix



Undirected Graph

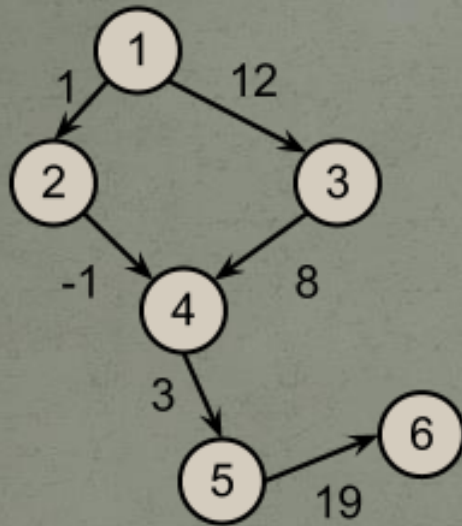
	1	2	3	4	5	6
1	0	1	1	0	0	0
2	1	0	0	1	0	0
3	1	0	0	1	0	0
4	0	1	1	0	1	0
5	0	0	0	1	0	1
6	0	0	0	0	1	0

Adjacency Matrix

<http://www.stoimen.com/>

# Graphs and adjacency matrices

## Weighted Directed Graph & Adjacency Matrix



Weighted Directed Graph

	①	②	③	④	⑤	⑥
①	0	1	12	0	0	0
②	-1	0	0	-1	0	0
③	-12	0	0	8	0	0
④	0	1	-8	0	3	0
⑤	0	0	0	-3	0	19
⑥	0	0	0	0	-19	0

Adjacency Matrix



# “Influence” based on centrality measures

- Degree Centrality
  - Measures the immediate rate of spread of a replicable commodity by a node
- Closeness Centrality
  - Average length of geodesic paths to all nodes in the network
- Betweenness Centrality
  - The number of geodesics on which a particular node lies

# Measuring the influence of a node in a social network

## Eigenvector Centrality (EVC)

- A node's "influence" is a function of its neighbors' influence
- Recursive definition
- Does not assume shortest path flow
- Assumes an "influence process" for the diffusion of a commodity through the network

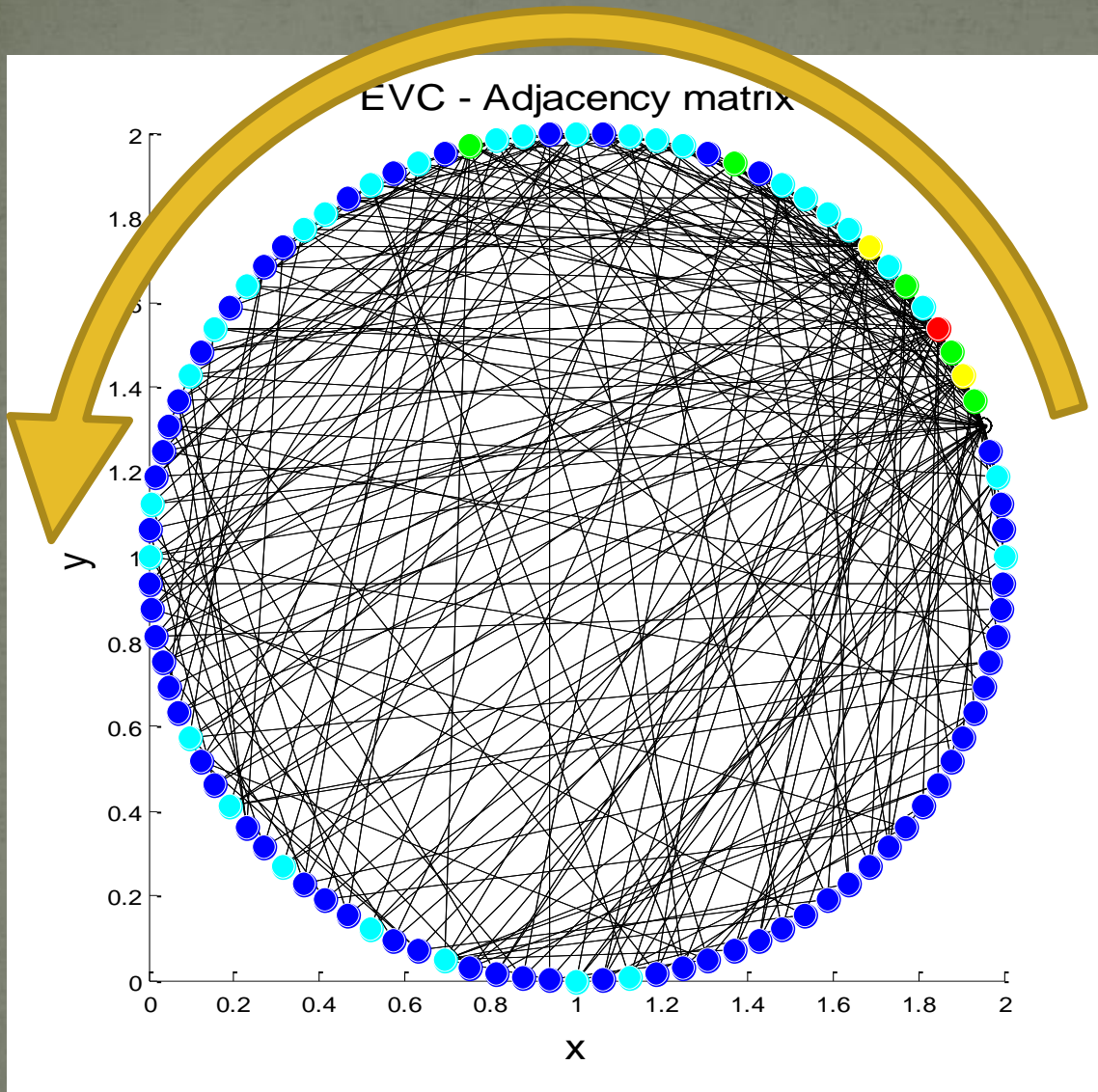
# Eigenvector Centrality

Scalar form      Vector form

$$\begin{aligned}x(i) &= \frac{1}{\lambda} \sum_{j \in \Gamma(v_j)} x(j) \\ &= \frac{1}{\lambda} \sum_{j=1}^N A_{i,j} x(j)\end{aligned}$$

$$\mathbf{x} = \frac{1}{\lambda} \mathbf{A} \mathbf{x}$$

$$\lambda \mathbf{x} = \mathbf{A} \mathbf{x}$$



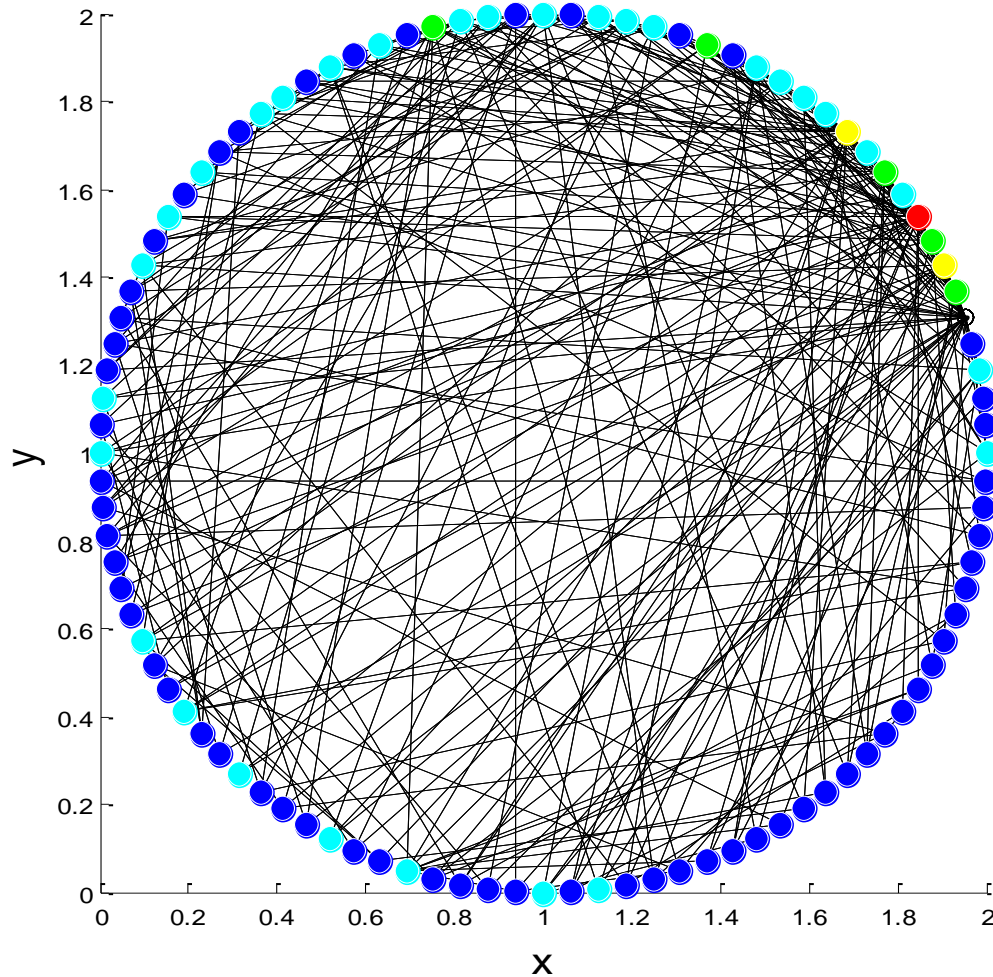
## EVC of 100 Node Barabasi-Albert Graph

- Node degree distribution follows a power law.
- In this drawing, node degrees go down as we move counter-clockwise on circle.

# Limitations of Eigenvector Centrality

- EVC works well enough in graphs consisting of a single cluster/community of nodes.
- When a graph has polarity and contains multiple communities the principal eigenvector is “pulled” in the direction of the largest community, away from other, smaller communities.
- Examples:
  - Social graphs capturing competing ideas/views/ideologies
  - Wireless networks
  - Other graphs with high clustering coefficients

## EVC - Adjacency matrix

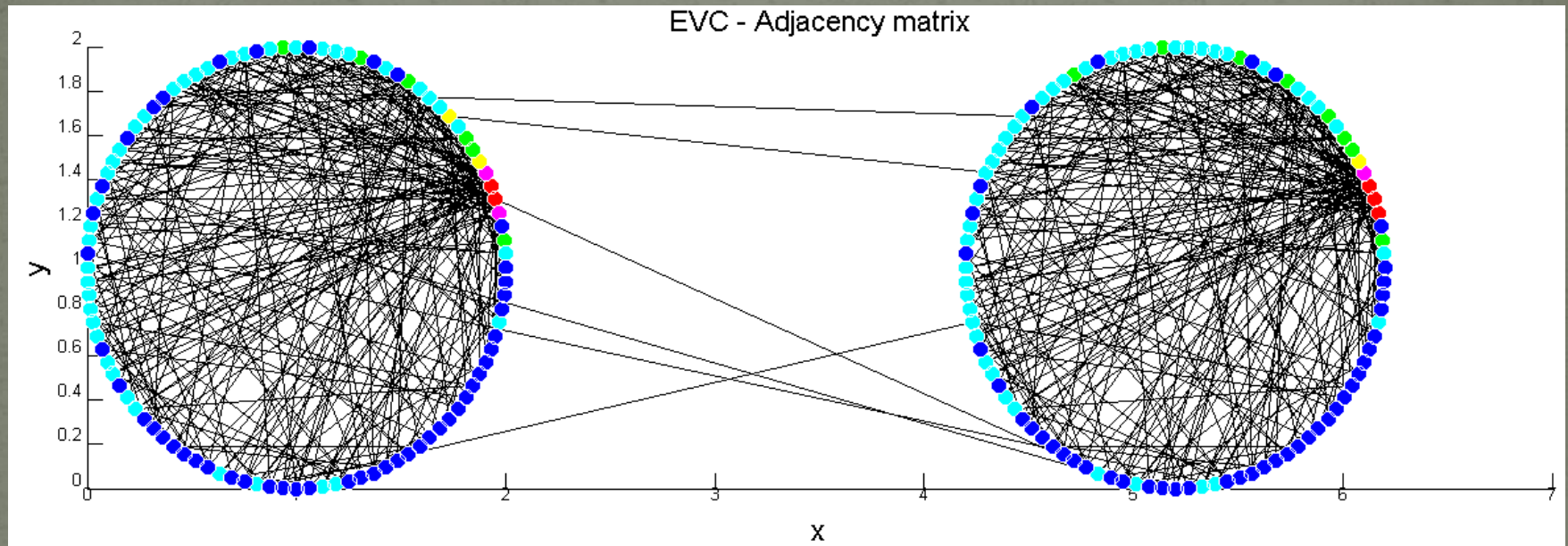


## EVC of 100 Node Barabasi-Albert Graph

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# Eigenvector Centrality

Weakly Connected Graphs: 100 + 100 Nodes

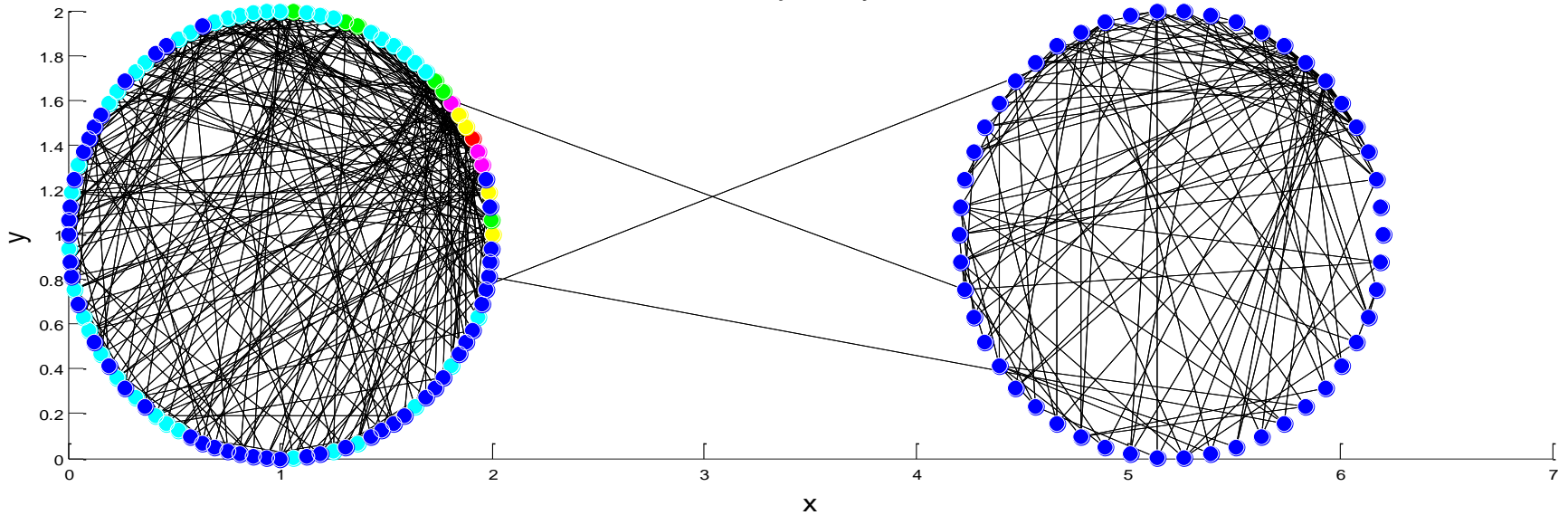


- The two subnets are copies of each other.
- Network consist of 100 + 100 nodes.
- EVC is able to identify the same nodes as “most central” in both networks.

# Eigenvector Centrality

## Weakly Connected Graphs: 100 + 50 Nodes

EVC - Adjacency matrix



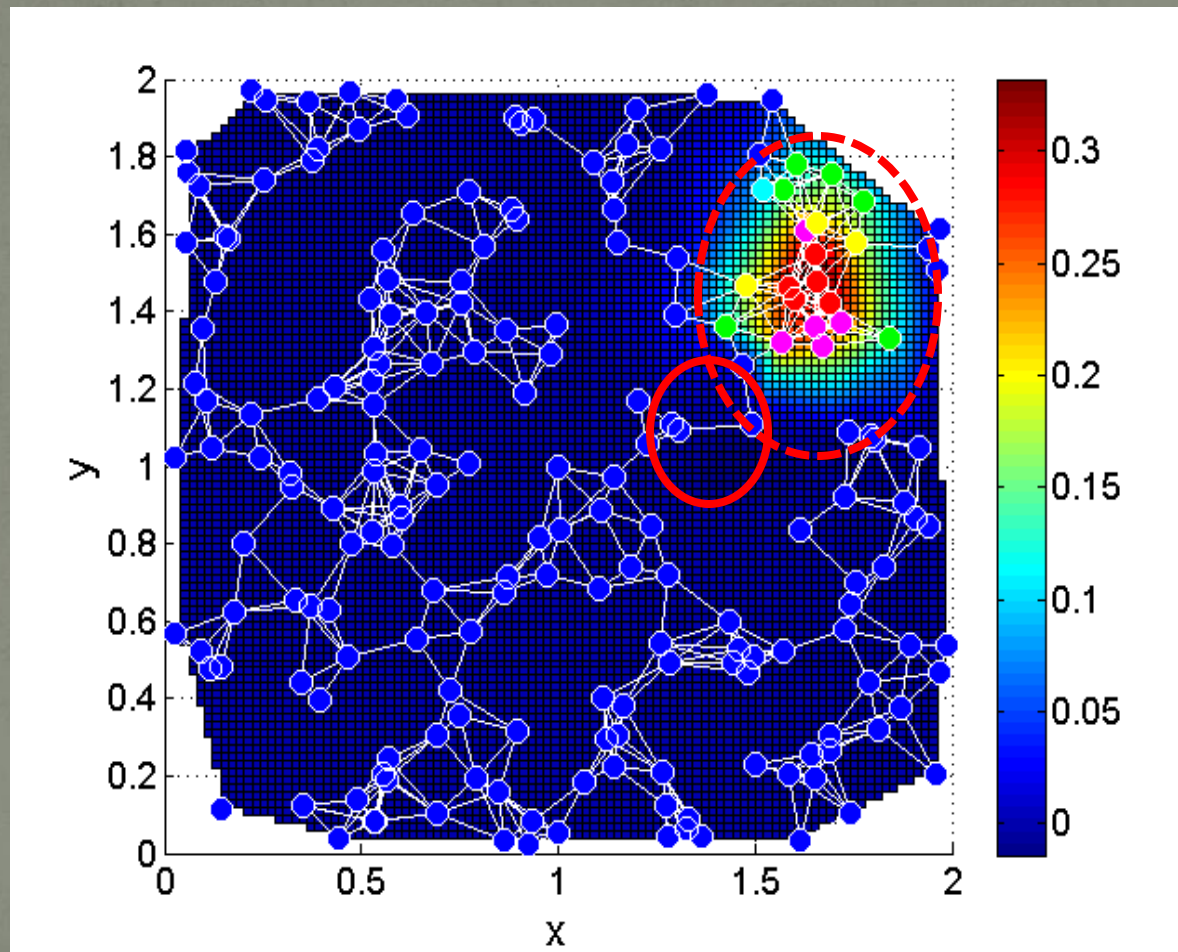
- The two subnets consist of 100 + 50 nodes.
- EVC assigns high centrality scores to nodes in the larger BA subnet, almost completely disregarding the smaller component.



# New Centrality Measure Needed?

- When dealing with complex massive networks with a large number of clusters, we need to search and examine a **multi-dimensional vector space** (in the overall spectral space of the network graph)
- An “influential” node could have its energy concentrated in one or more of the dimensions of the multi-dimensional vector space

# Eigenvector Centrality of Mesh Network



# Principal Component Centrality

## Principal Component Centrality (PCC)

- Measured using multiple eigenvectors in a  $P$ -dimensional spectral space of a graph
- A node's PCC is the  $\ell_2$  norm of its coordinates in the  $P$ -dimensional hyperspace formed by the  $P$  most significant eigenvectors as its basis.

Muhammad U. Ilyas and Hayder Radha , "A KLT-inspired Node Centrality for Identifying Influential Neighborhoods in Graphs," *Proceedings of the 44th Conference on Information Sciences and Systems (CISS'10)*, Princeton Univ., March 17, 2010

# Principal Component Centrality

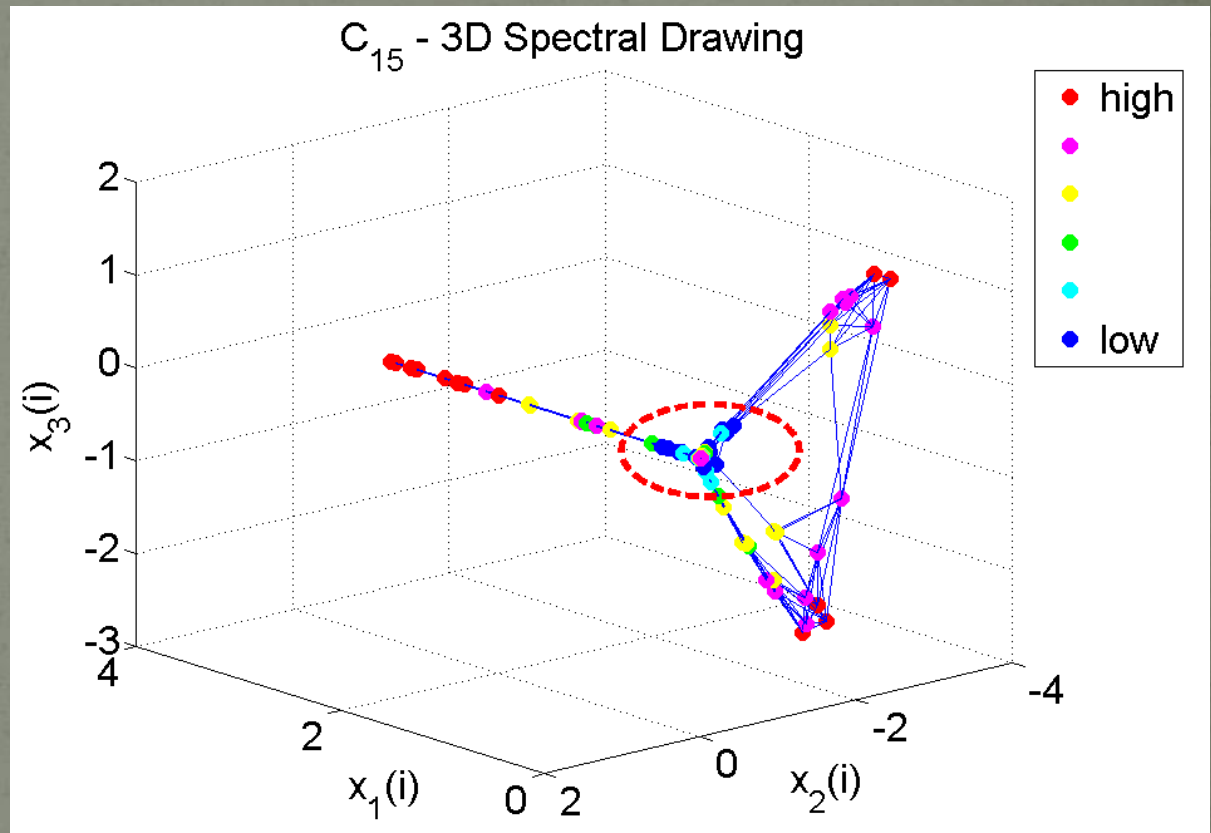
## Matrix Formulation

$$\begin{aligned} \mathbf{C}_P &= \sqrt{\left( (\mathbf{A}\mathbf{X}_{N \times P}) \circ (\mathbf{A}\mathbf{X}_{N \times P}) \right) \mathbf{1}_{P \times 1}} \\ &= \sqrt{\left( \mathbf{X}_{N \times P} \circ \mathbf{X}_{N \times P} \right) \left( \mathbf{\Lambda}_{P \times 1} \circ \mathbf{\Lambda}_{P \times 1} \right)} \end{aligned}$$

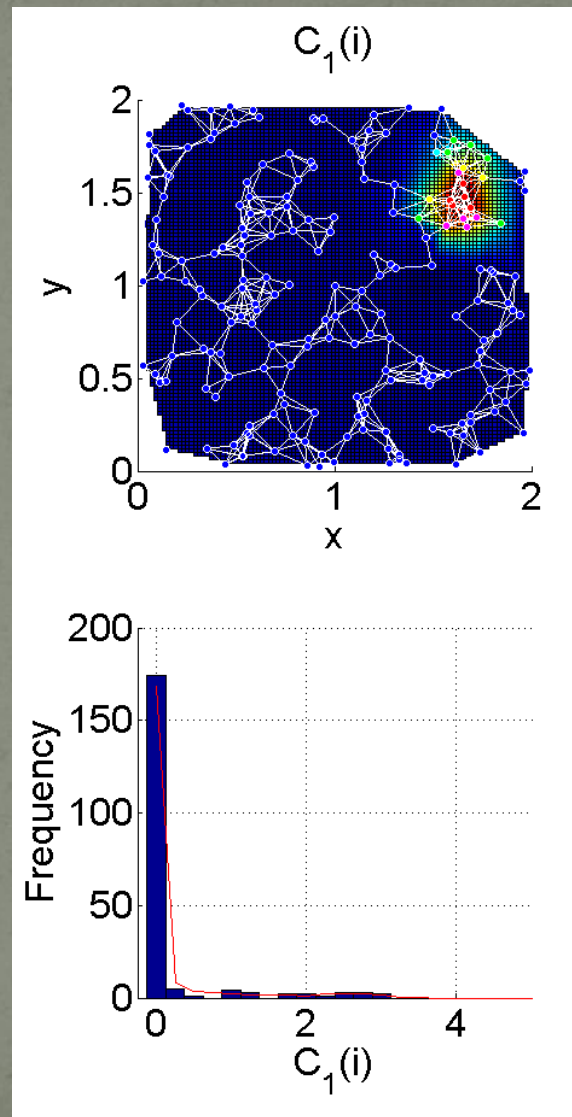
- The Hadamard/ Schur/ entrywise product operator is used

# Graphical Interpretation of PCC

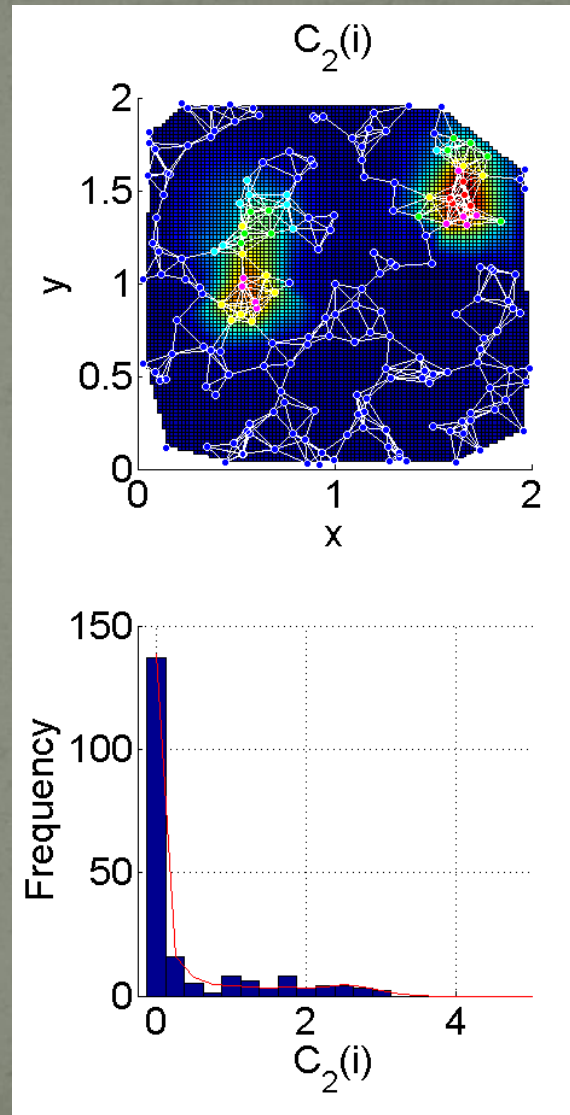
- Spectral drawing of mesh graph in 3 dimensions
- Nodes are positioned based on first 3 eigenvectors.
- Nodes are colored according to  $C_{15}$  (15 feature PCC).



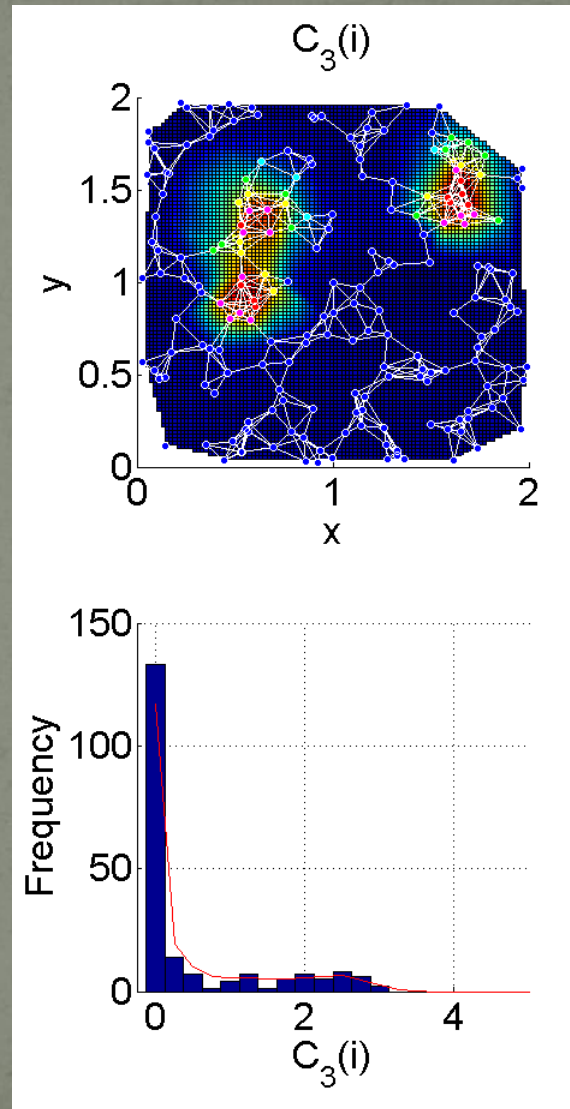
$P=1$



$P=2$

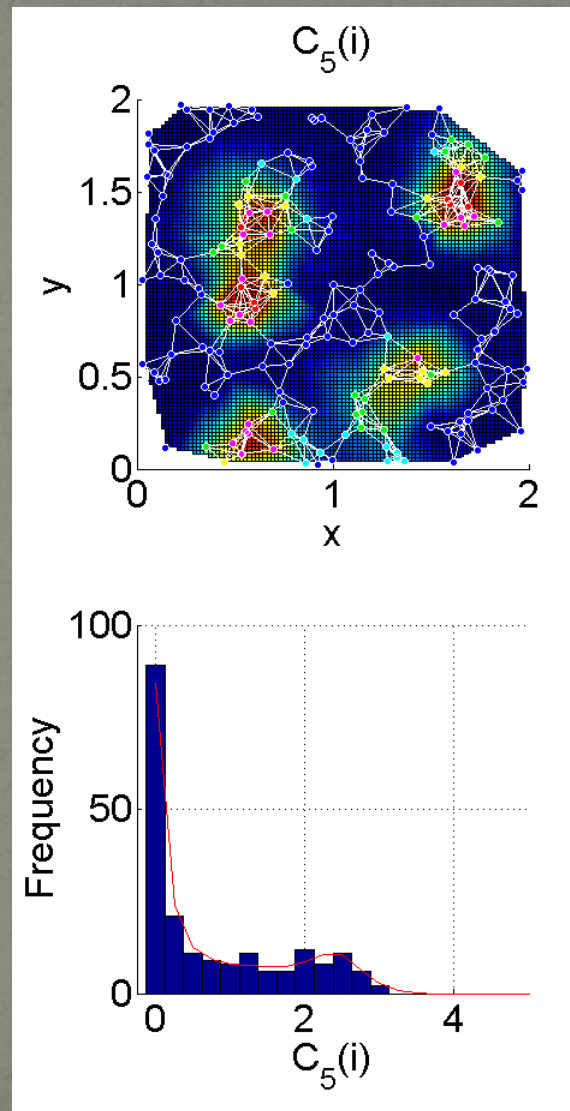


$P=3$

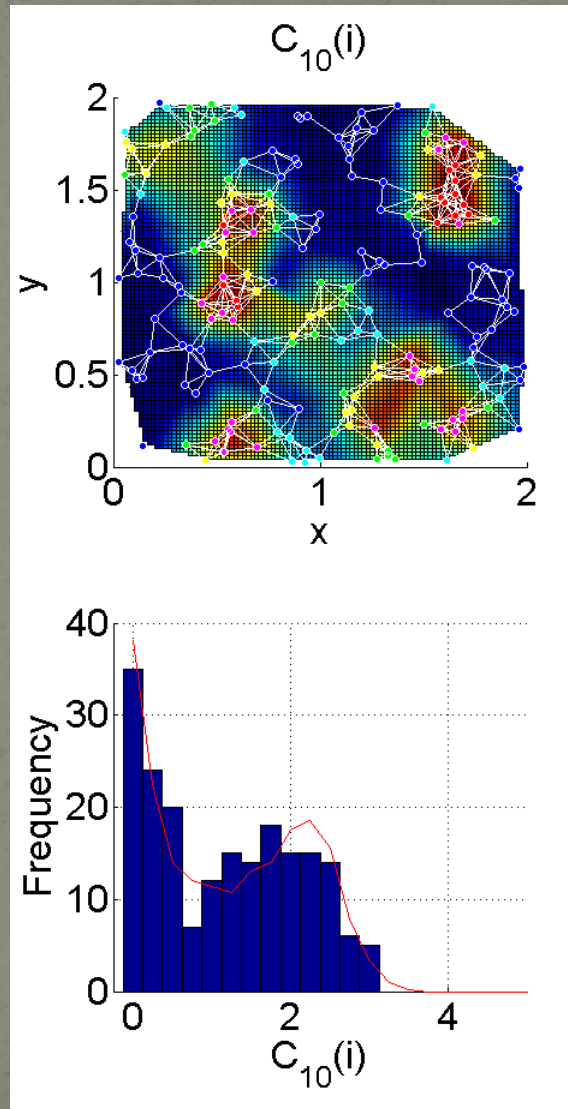




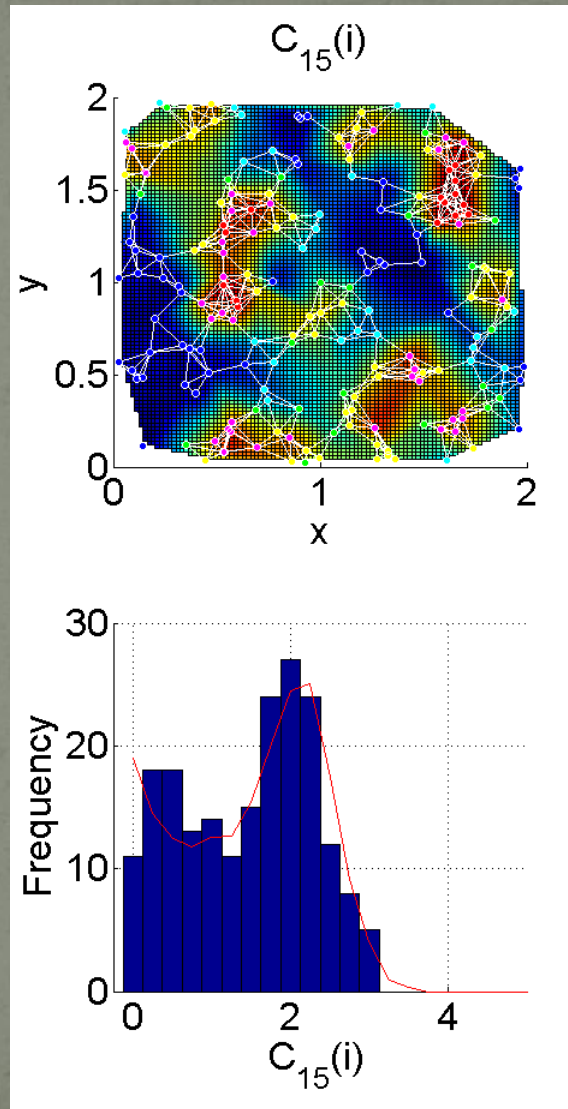
$P=5$



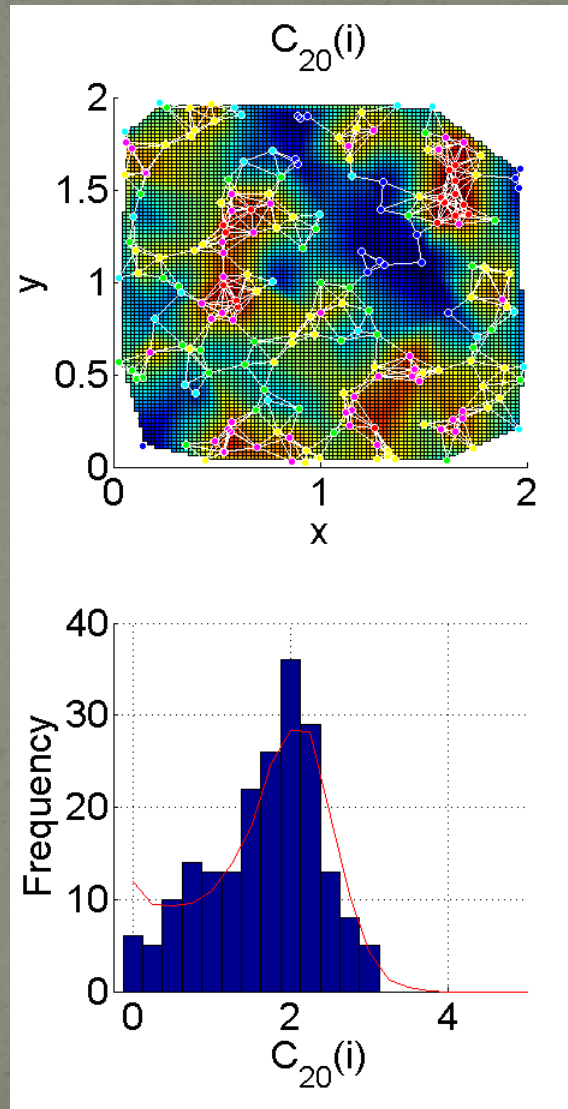
$P=10$



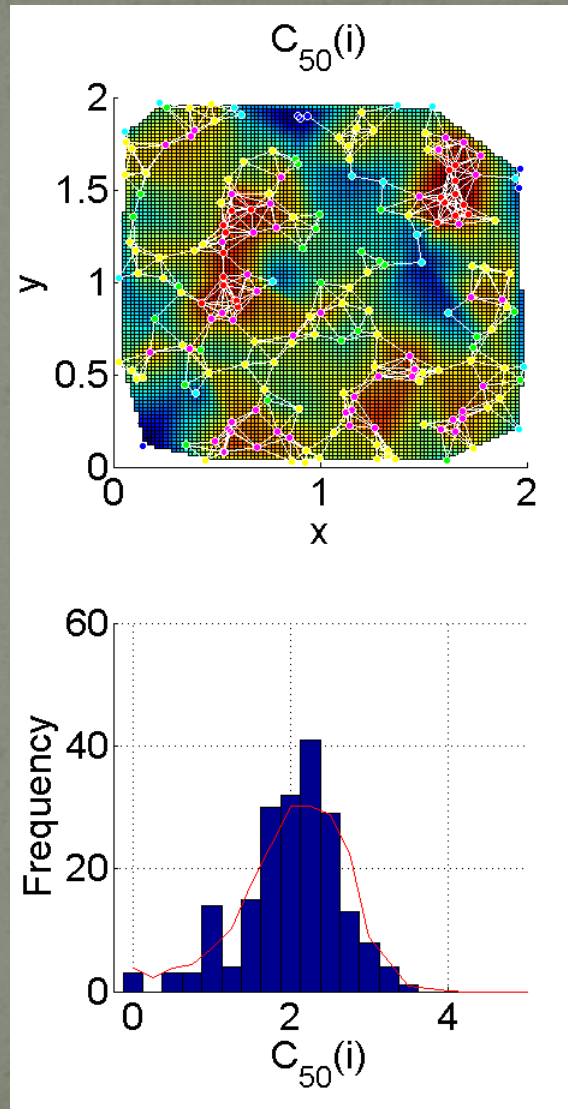
$P=15$



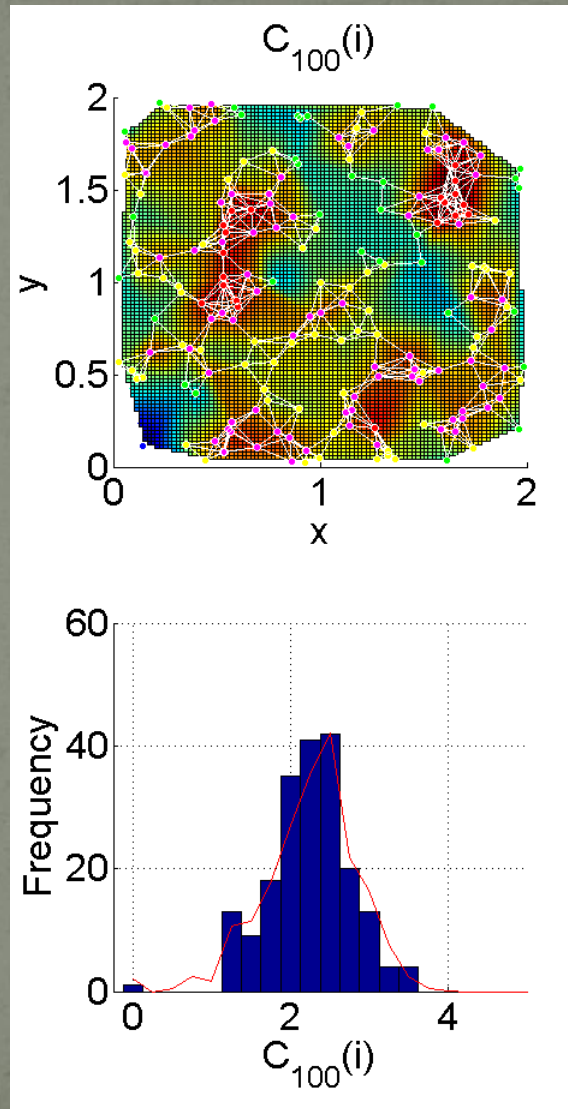
$P=20$



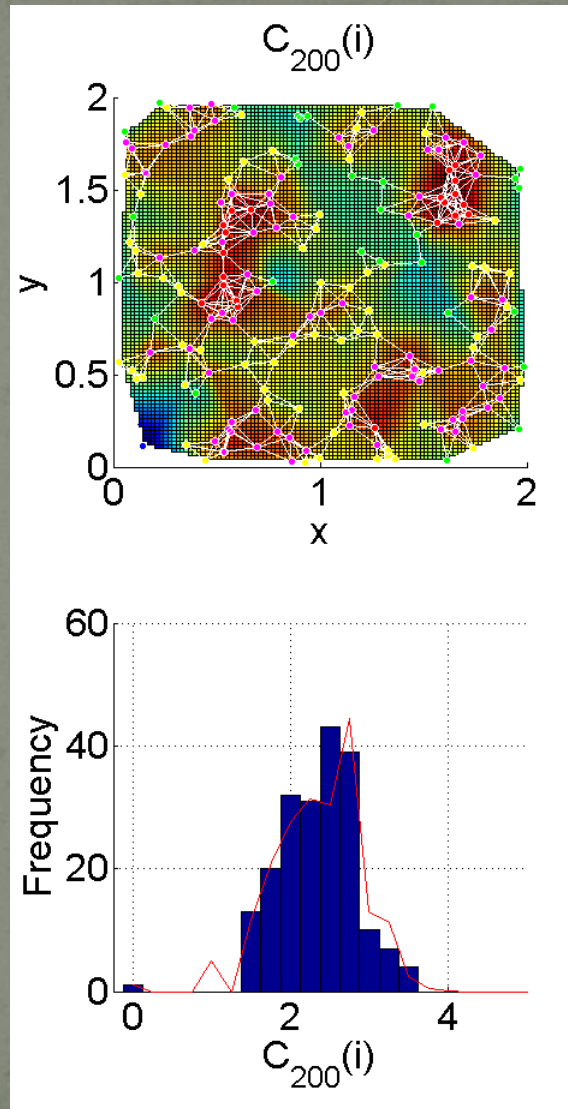
$P=50$



$P=100$

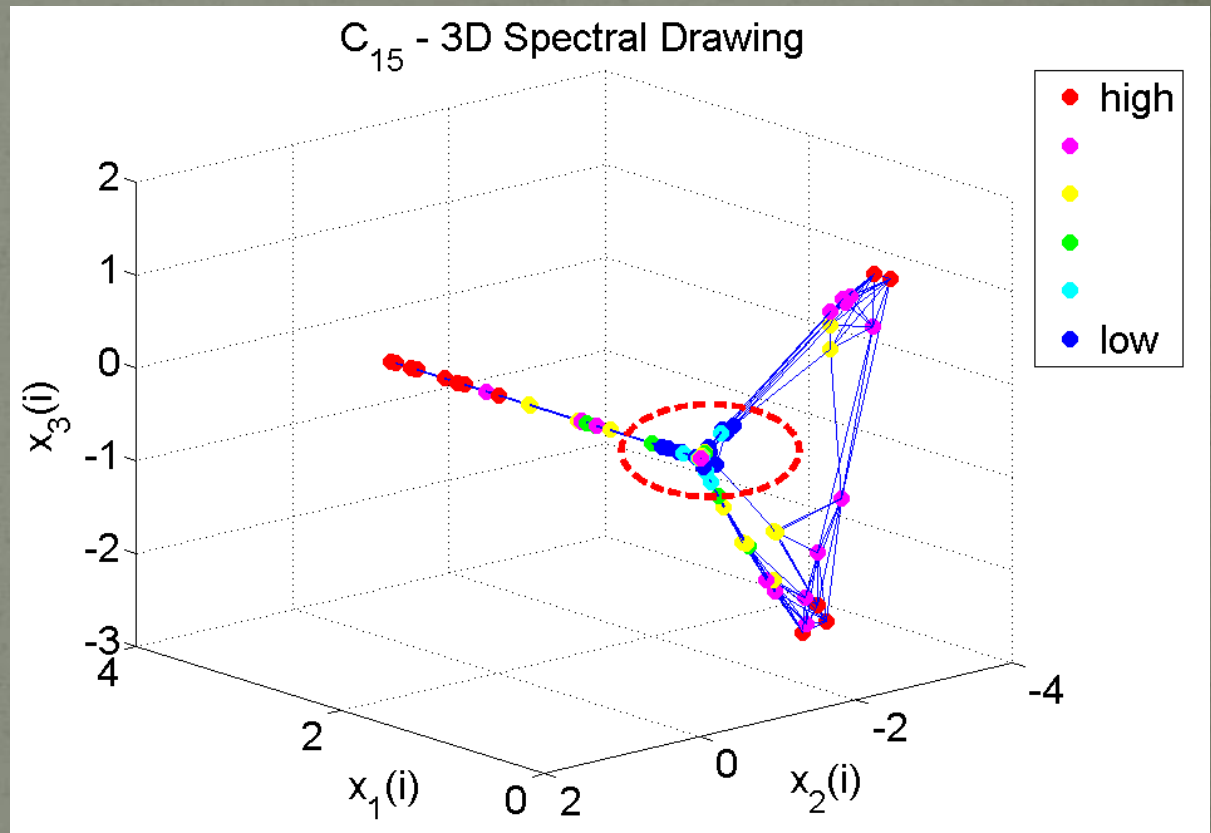


$P=200$



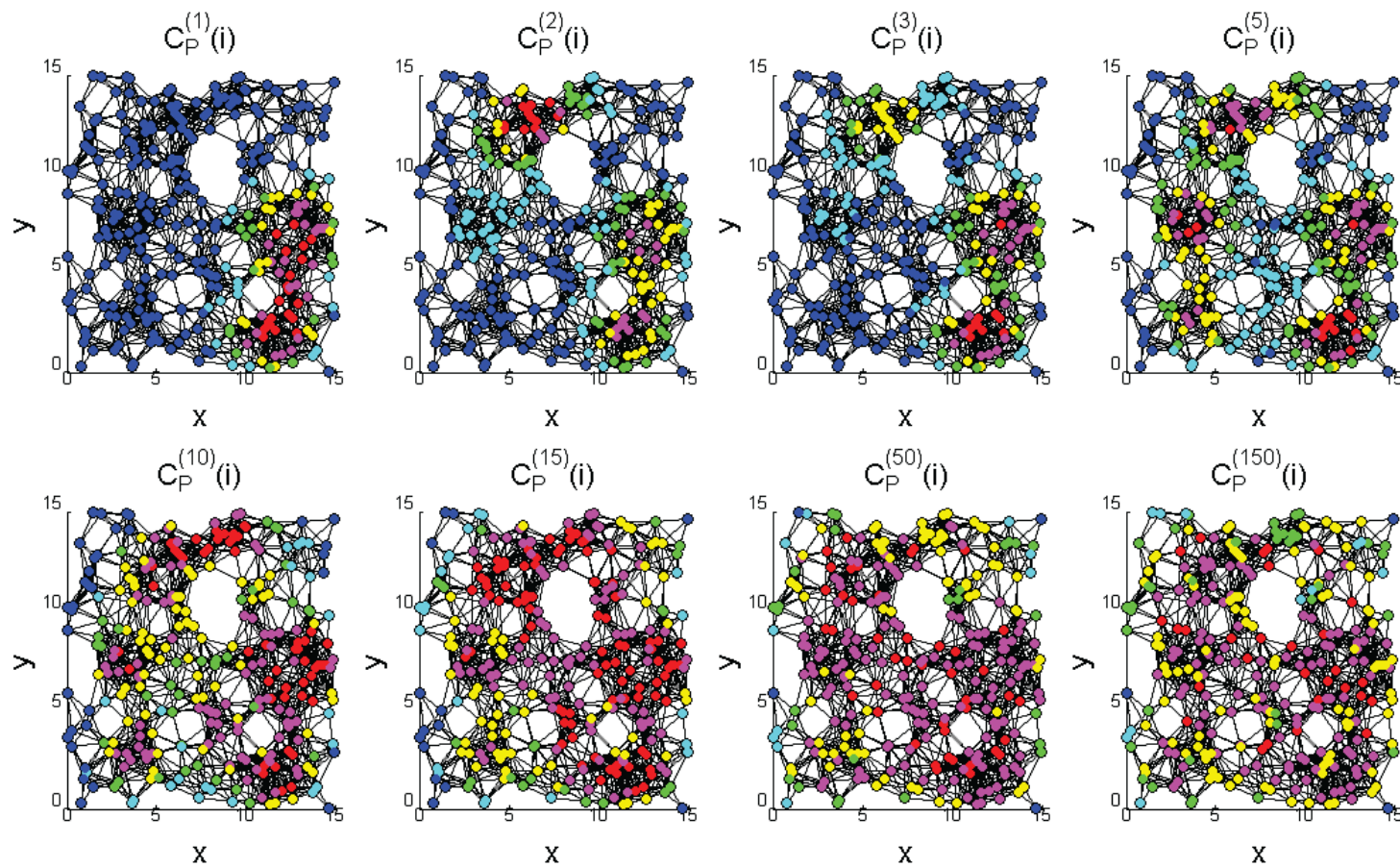
# Graphical Interpretation of PCC

- Spectral drawing of mesh graph in 3 dimensions
- Nodes are positioned based on first 3 eigenvectors.
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# Measuring influence using Principle Component Centrality (PCC)



# How many "influential" nodes?

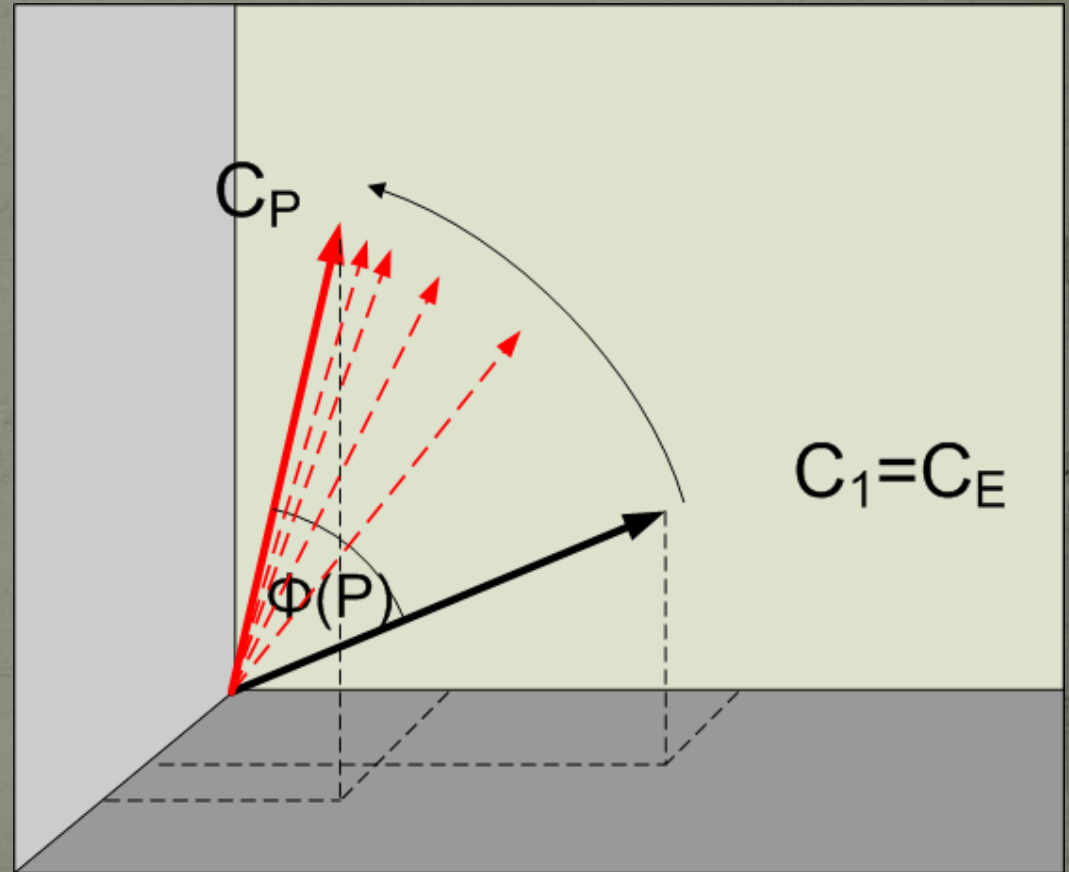
- What criteria should be used to choose an appropriate number of “features” for PCC?
- Time and space complexity of eigendecomposition is significant
- Prefer to compute PCC with fewer eigenvectors if possible

# Criteria for Feature Selection: PCC-EVC Phase Angle

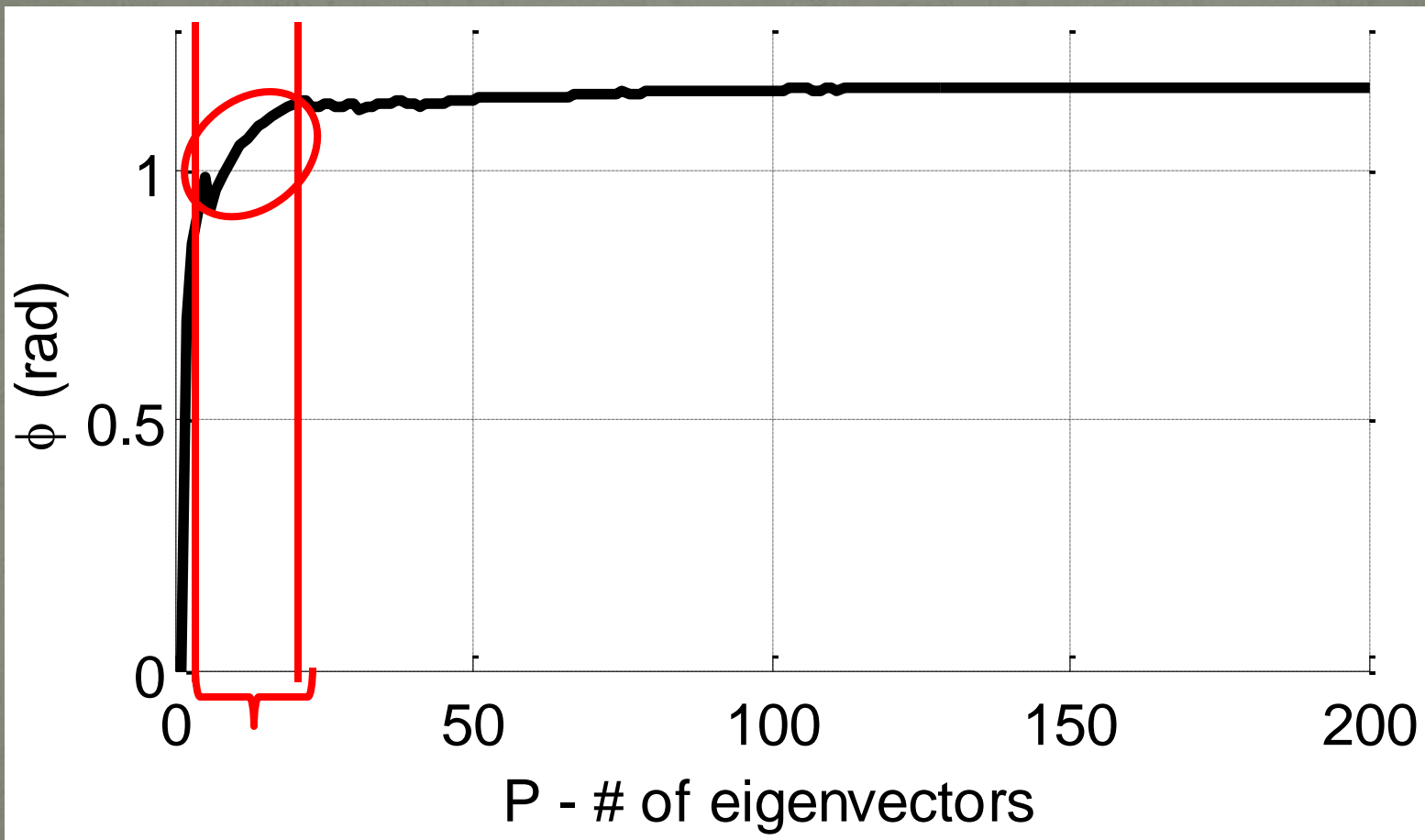
In N-dimensional hyperspace of centrality vectors,

- Compute phase angle between N-dimensional EVC and PCC vectors.
- Add another feature → recompute phase angle.

$$\phi(P) = \arccos \left( \frac{C_P \cdot C_E}{|C_P| |C_E|} \right)$$



# Phase Angle PCC vs EVC Vectors



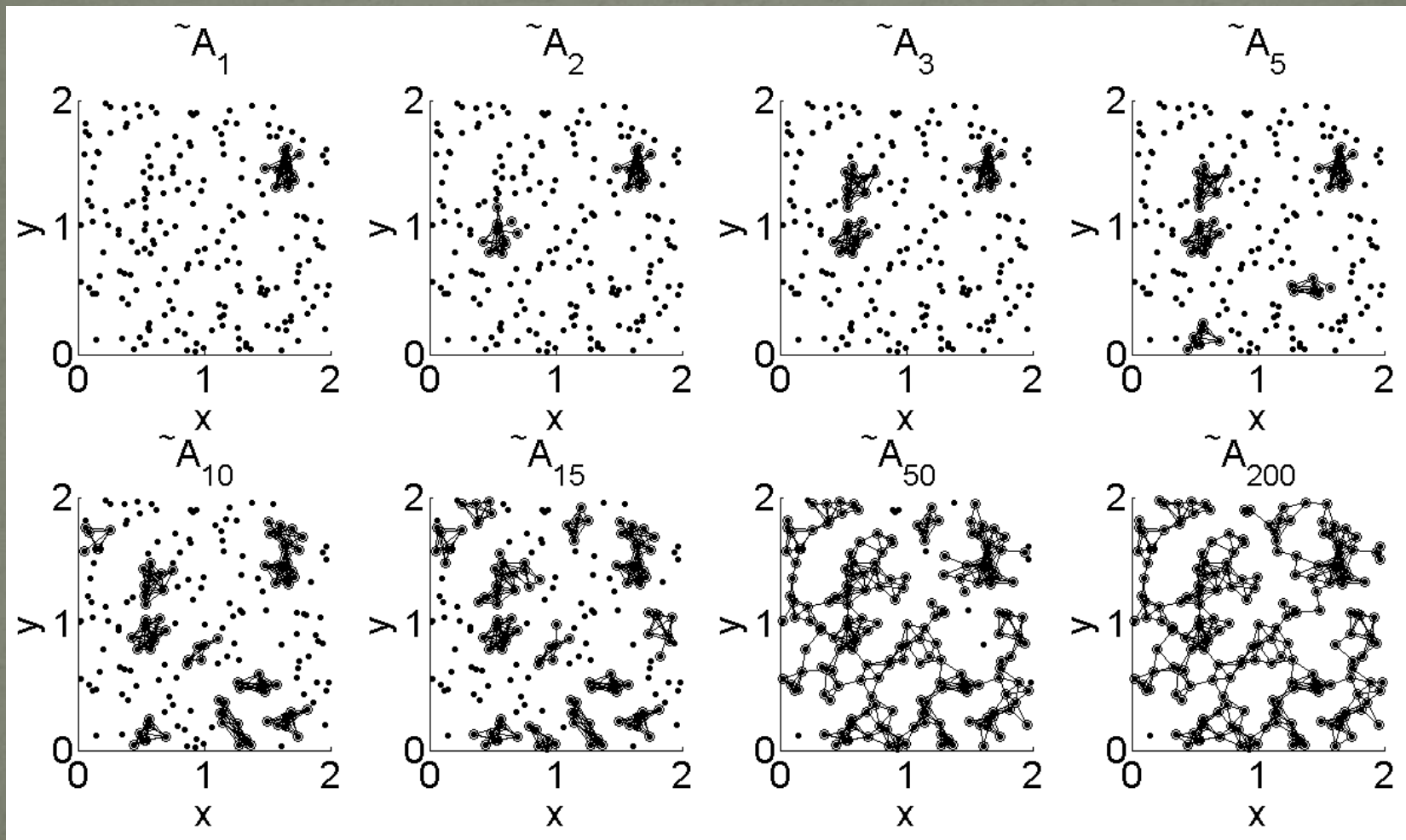
# Graph Reconstruction – Inverse Graph Transform

Graph's adjacency matrix can be reconstructed using its constituents eigenvectors components.

Partial reconstruction can be attempted using subset of features.

$$\mathbf{A}_P = \mathbf{X}_{N \times P} \mathbf{\Lambda}_{P \times P} \mathbf{X}_{P \times N}^T$$

# Graph Reconstructions



# Related Problem Areas

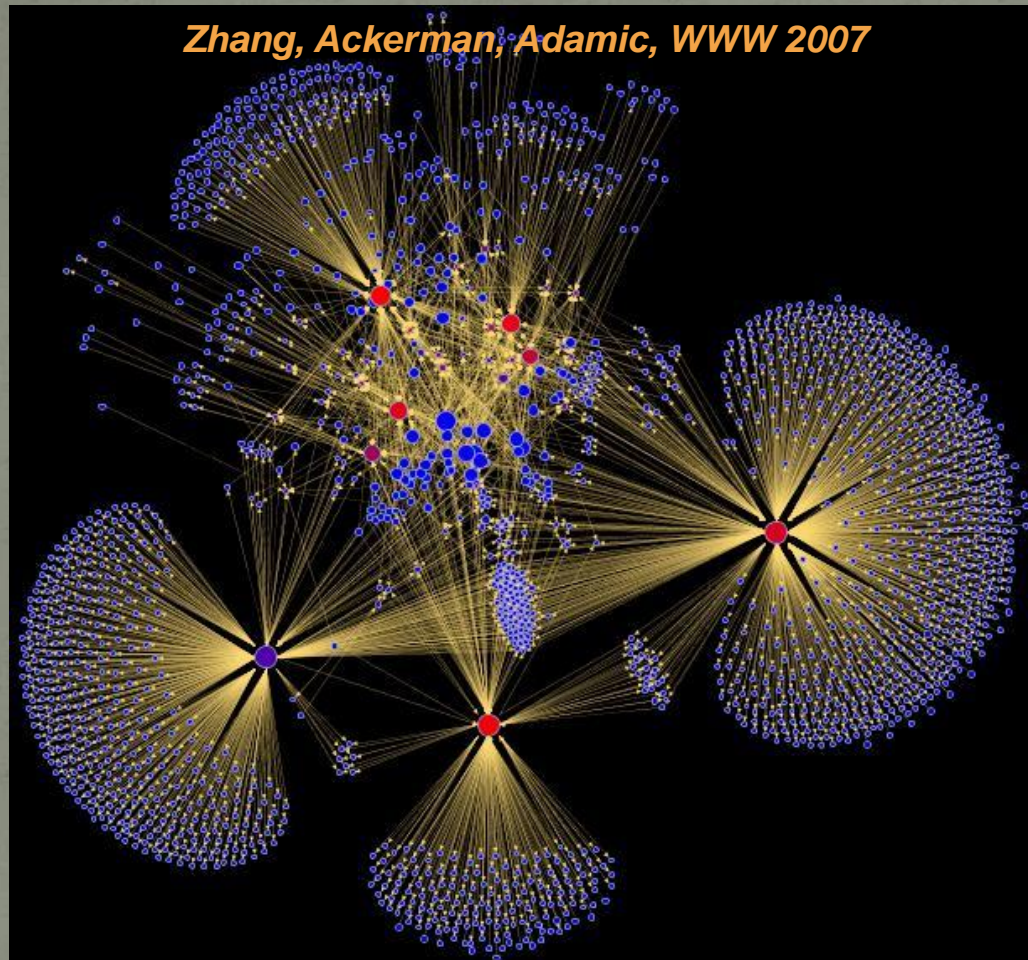
- Can we identify more specific roles of individuals in massive social networks?
  - Leaders versus followers?
  - What types of leaders?
- What can be learned about the role of “individual” *links* among nodes?
  - How important *each link* to the overall network?
  - Can this be used for “denoising” massive networks?
- The interaction between users and content in multimedia social networks such as YouTube

# Related Problem Areas

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# Identifying experts

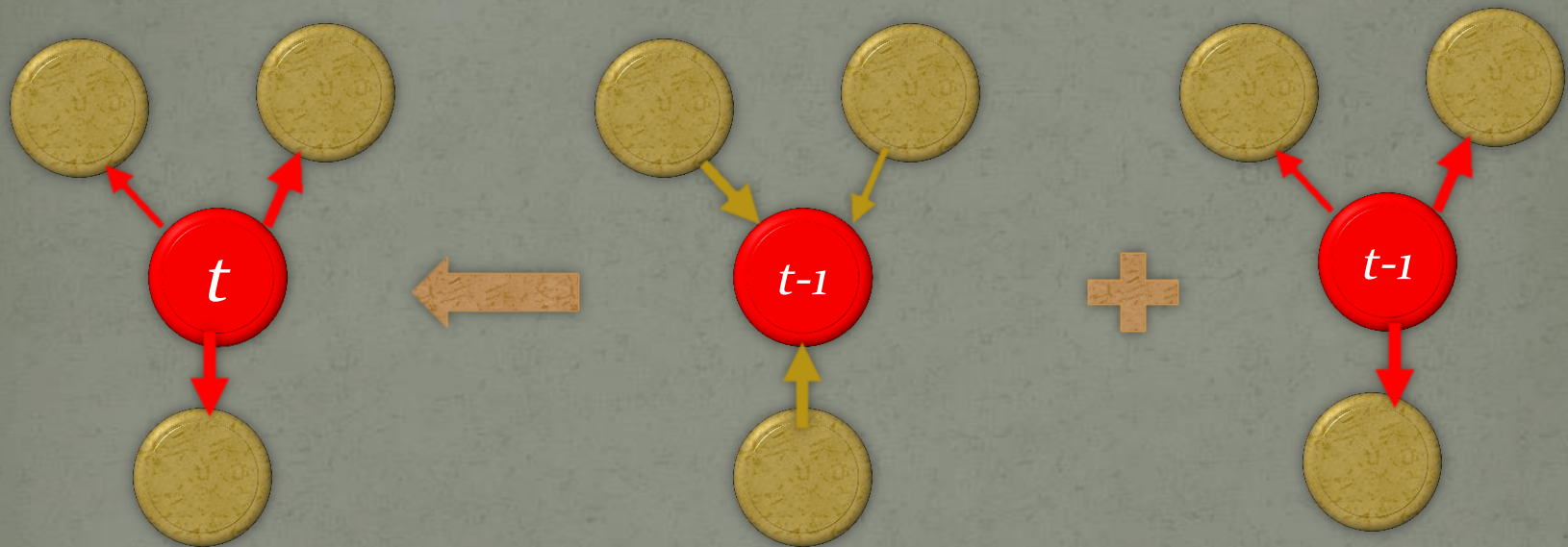


Portion of the Java Forum Q&A network

# Leaders vs. Followers

- Leaders' opinions are highly influential
- **Advertising** companies gain by giving free samples to “leaders” instead of a random population
- For **community health** campaigns, targeting interventions at community leaders have been shown to be more effective than applying them to random individuals
- For **administrative science**, identifying leaders results in effective product development teams with better work performance

# Friedkin-Johnsen Influence Model



- N. Friedkin and E. Johnsen. Social influence and opinions. The Journal of Mathematical Sociology, 1990.
- N. Friedkin and E. Johnsen. Social influence networks and opinion change. Advances in Group Processes, 1999.

# LUCI model for leaders and followers

- Outward interaction is influenced by external influences and own prior interactions
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$$y_i(t) = \rho_i(t) \sum_{\tau=1}^{\tau_{\max}} \sum_{j=1}^N m_{j,i}(t - \tau) + \gamma_i(t) \sum_{\tau=1}^{\tau_{\max}} \sum_{j=1}^N m_{i,j}(t - \tau) + e_i(t)$$

**Outward interaction** = **External influence** + **Own (history) influence**

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$$\min_{\rho_i(t), \gamma_i(t)} \left( y_i(t) - \left( \rho_i(t) \sum_{\tau=1}^{\tau_{\max}} \sum_{j=1}^N m_{j,i}(t-\tau) + \gamma_i(t) \sum_{\tau=1}^{\tau_{\max}} \sum_{j=1}^N m_{i,j}(t-\tau) \right) \right)^2$$

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$$y_i(t) = \sum_{j=1}^N m_{i,j}(t) = \rho_i \underbrace{\sum_{j=1}^N m_{j,i}(t-1)}_{\text{External influence}} + \gamma_i \underbrace{\sum_{j=1}^N m_{i,j}(t-1)}_{\text{Own (history) influence}} + e_i$$

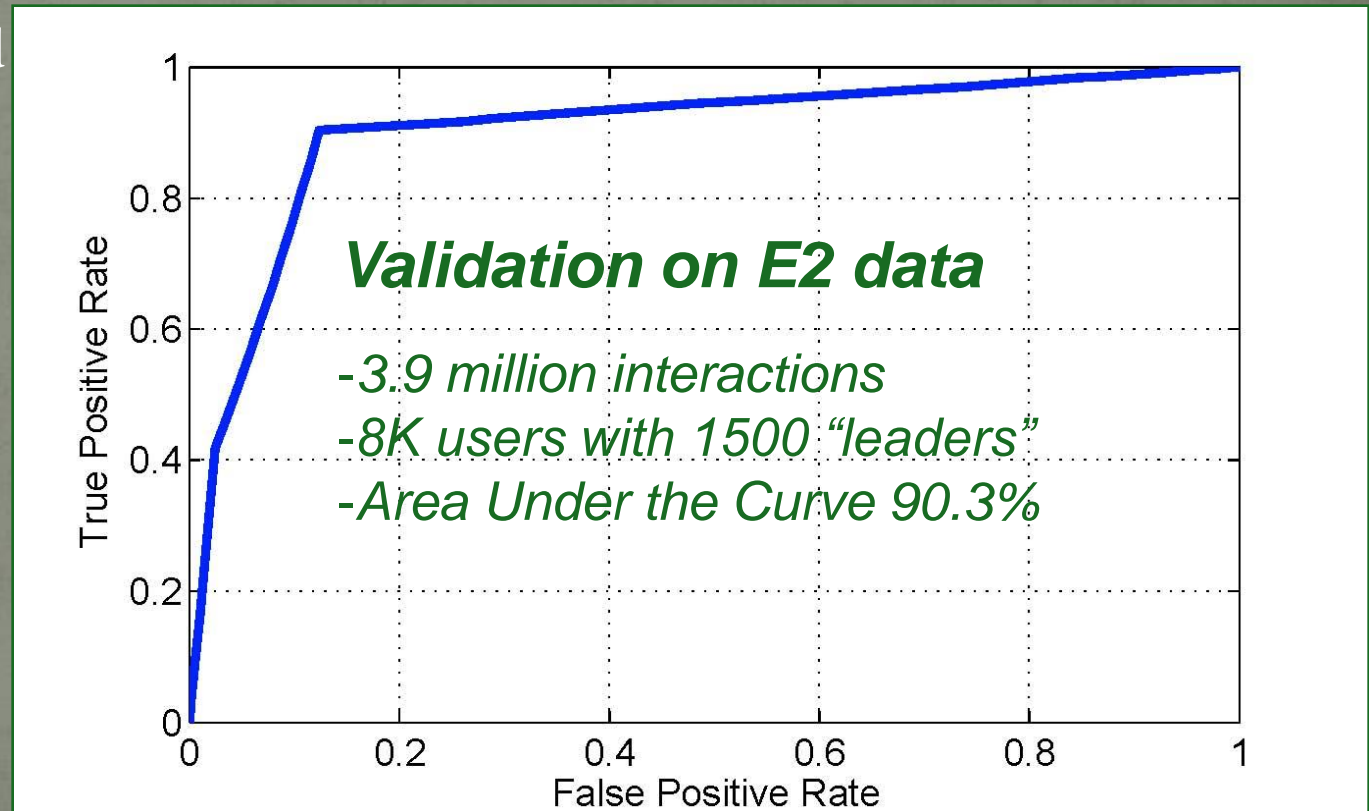
**Outward interaction** = **External influence** + **Own (history) influence**

# LUCI model for leaders and followers

- Everything<sub>2</sub>, or E<sub>2</sub>, “a collaborative Web-based community consisting of a database of interlinked user-submitted written material.”

In 2006: hosted by U. Michigan Ann Arbor. “We exist thanks to their generosity” (which is motivated by their academic curiosity, I suppose).”

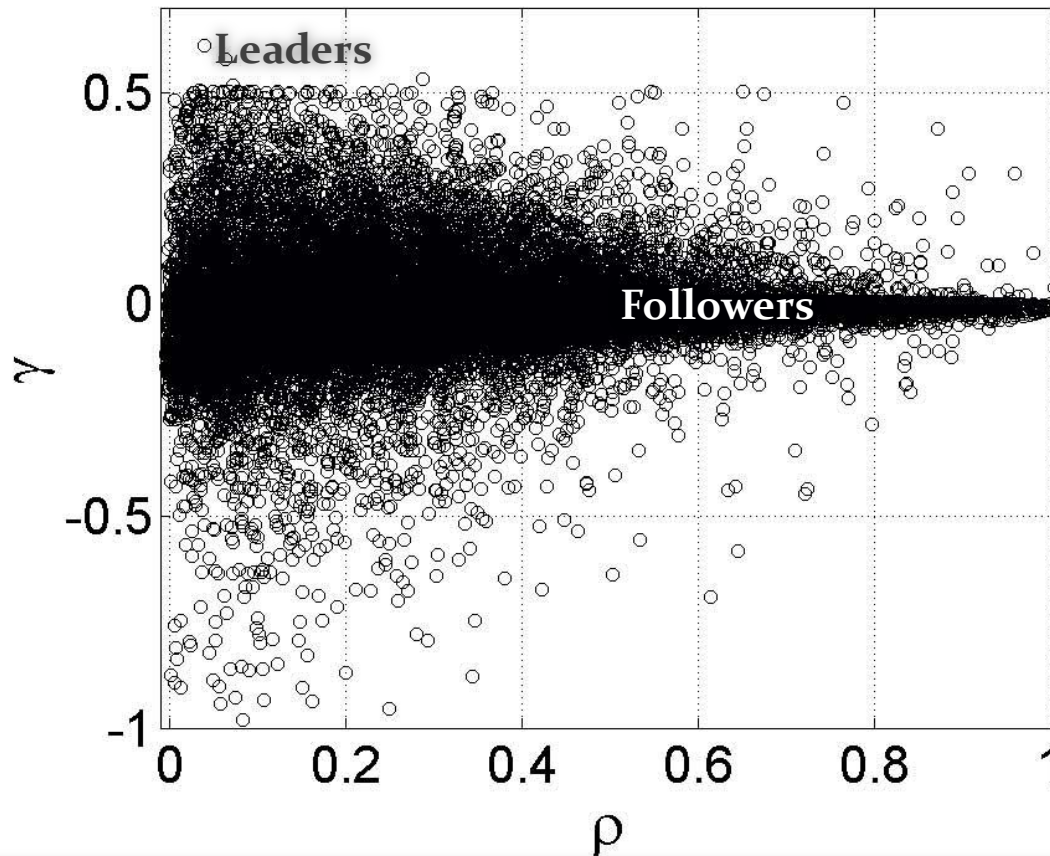
E<sub>2</sub> servers moved to MSU in 2007





# LUCI model for leaders and followers

Own history influence coefficient



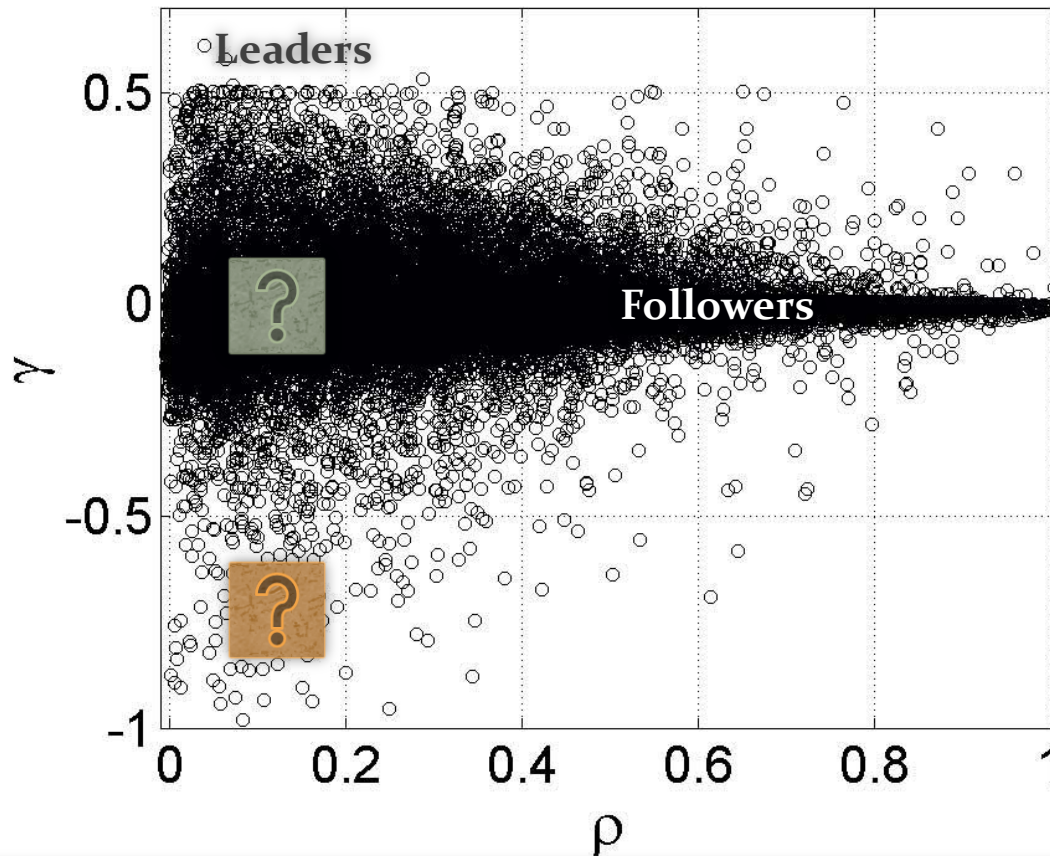
External influence coefficient

## Facebook data

- 3 million users
- 23 million edges
- Interaction data over one year
- Time sample ( $t$ ) is one month

# LUCI model for leaders and followers

Own history influence coefficient



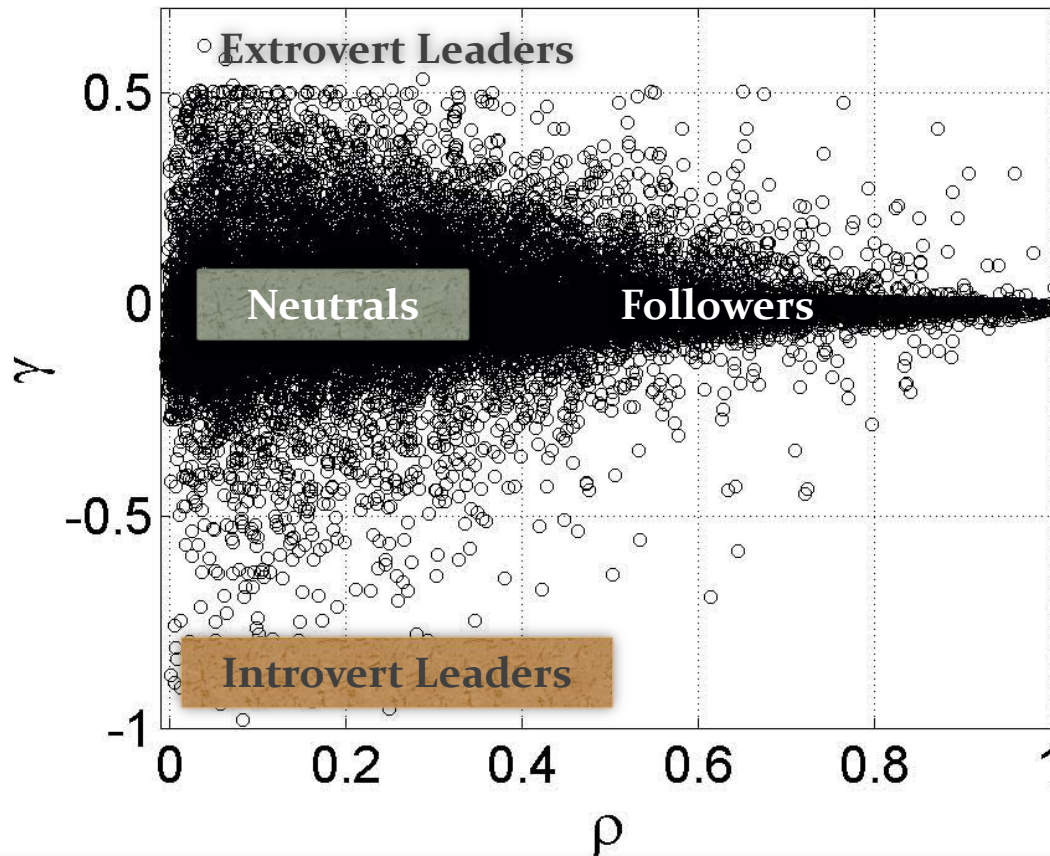
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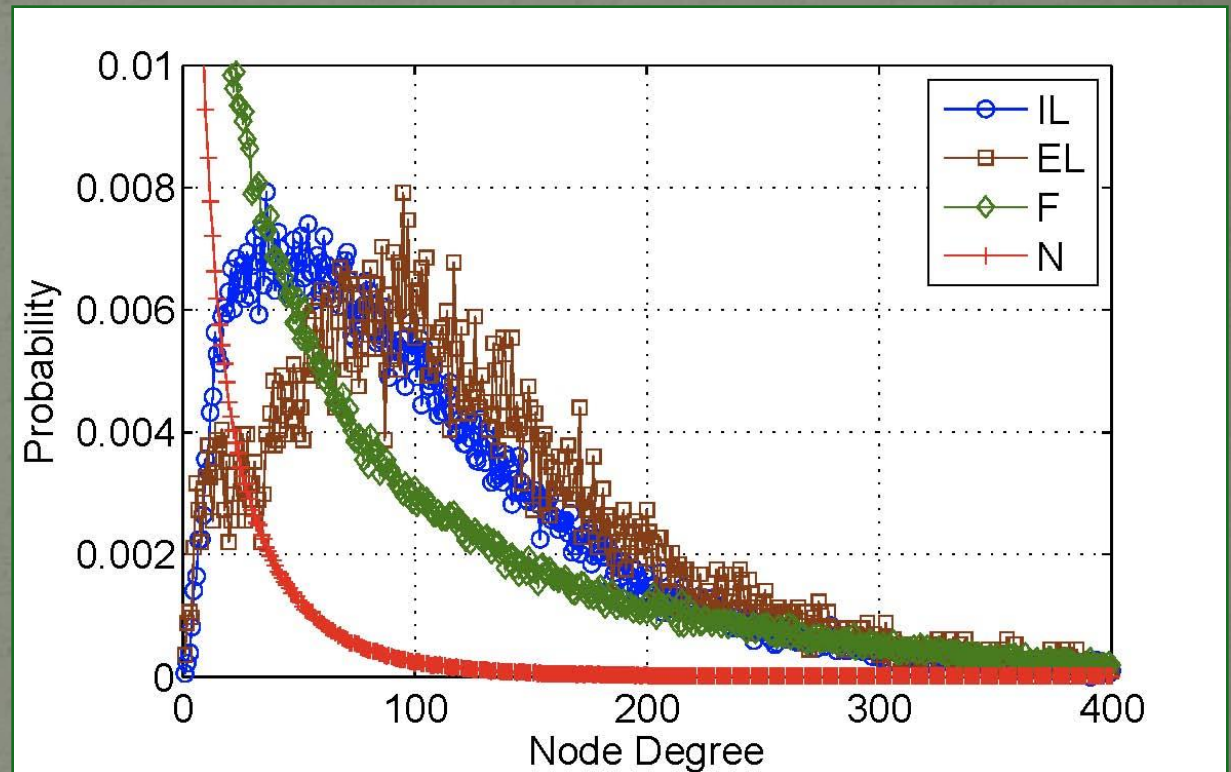
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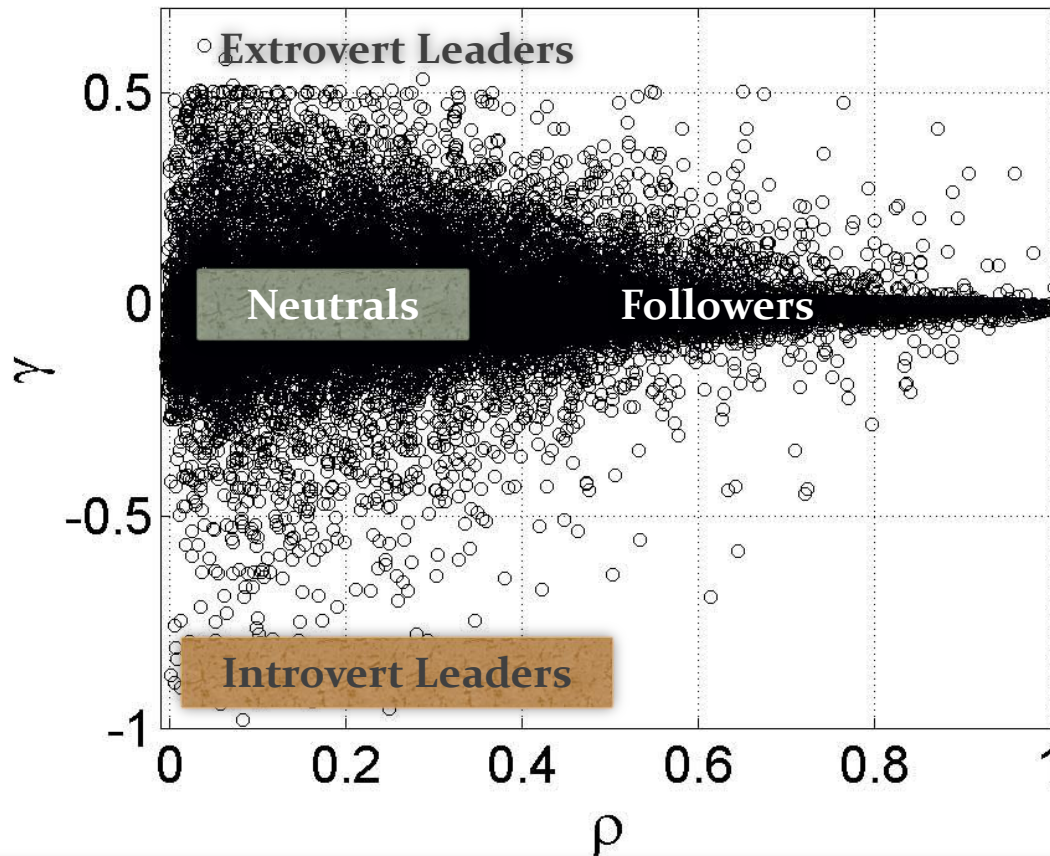
# Leaders versus non-leaders

Degree distributions for leaders are quite different from the degree distributions for non-leaders



# LUCI model for leaders and followers

Own history influence coefficient



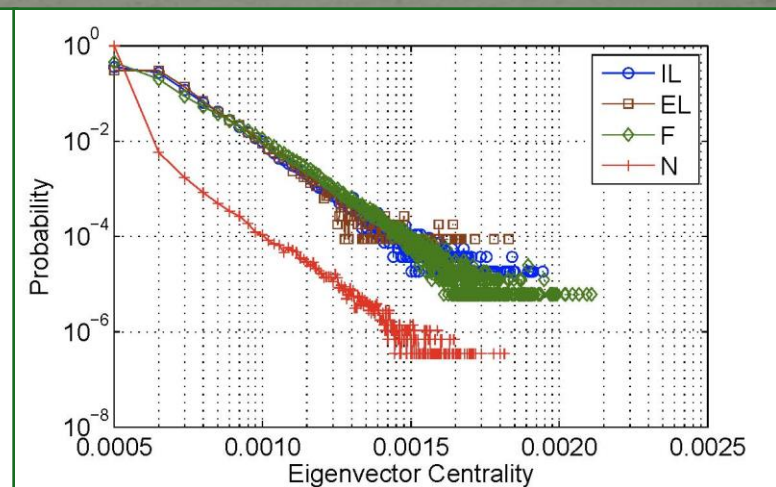
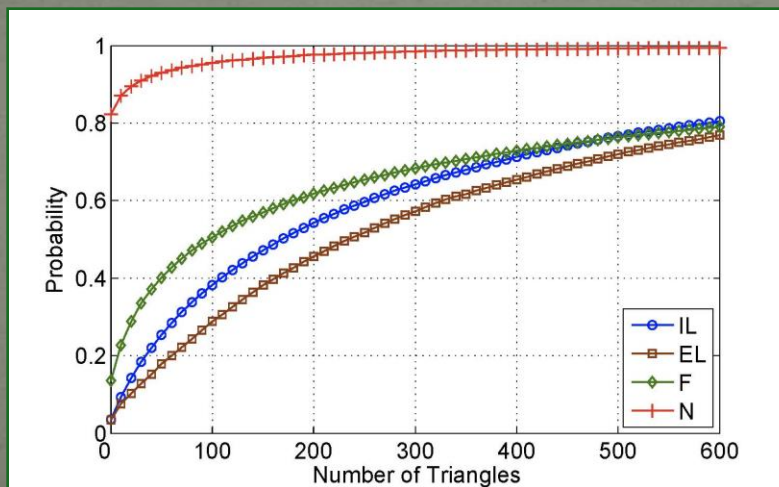
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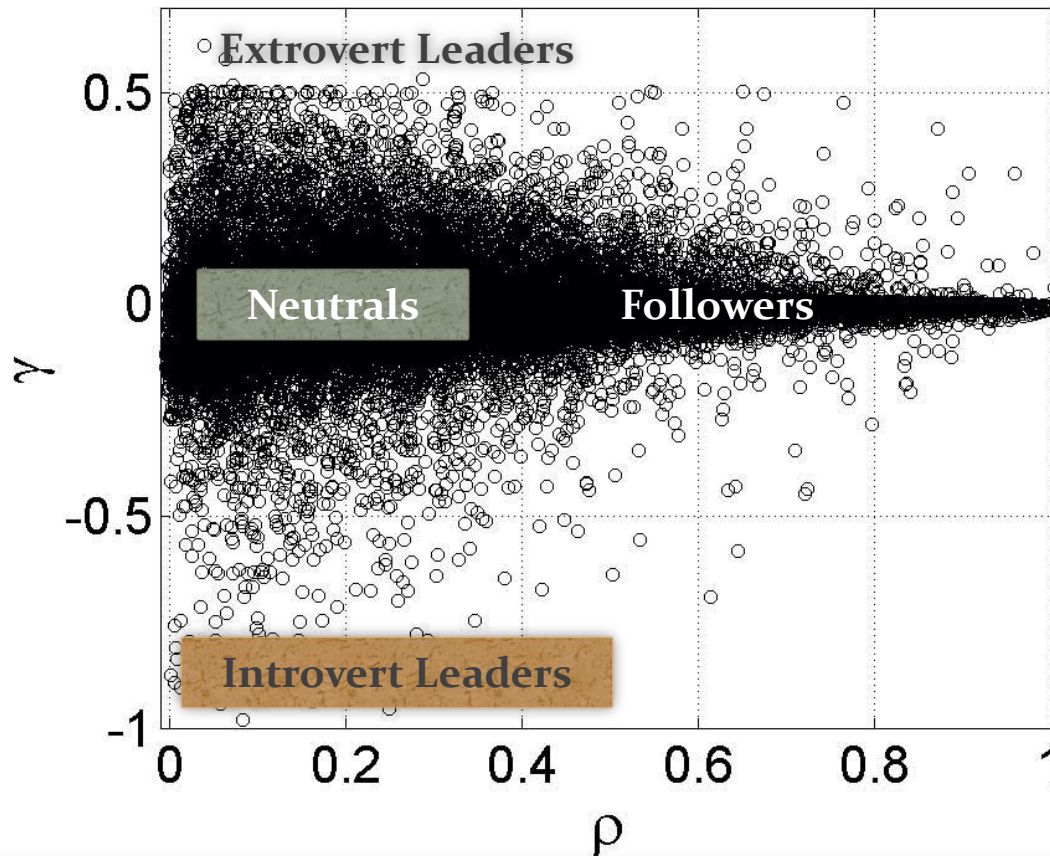
# Neutrals are different

- **Neutrals'** levels of interaction are independent of interaction levels of their friends
- Have the lowest average degree and are mostly connected to followers or other neutrals in the friendship graph
- The average eigenvector centrality of neutrals is two orders of magnitude lower than other user categories



# LUCI model for leaders and followers

Own history influence coefficient



External influence coefficient

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# Introvert Leaders

- “... 40% of executives describe themselves as introverts, including Microsoft’s **Bill Gates**, the über-investors **Warren Buffett** and **Charles Schwab**, ... .Odds are President **Barack Obama** is an innie as well. What does that mean? That introverts, not just extroverts, have the right stuff to lead organizations in a go-go, extroverted business culture”

*Forbes: **Why Introverts Can Make The Best Leaders**, Nov. 2009, Jennifer B. Kahnweiler*

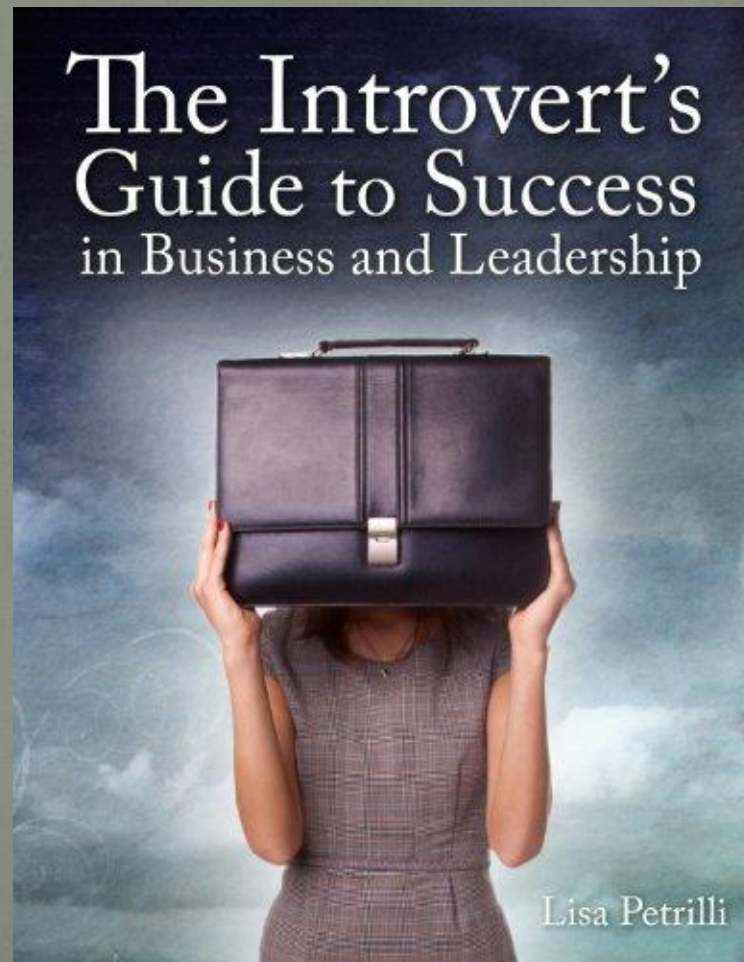
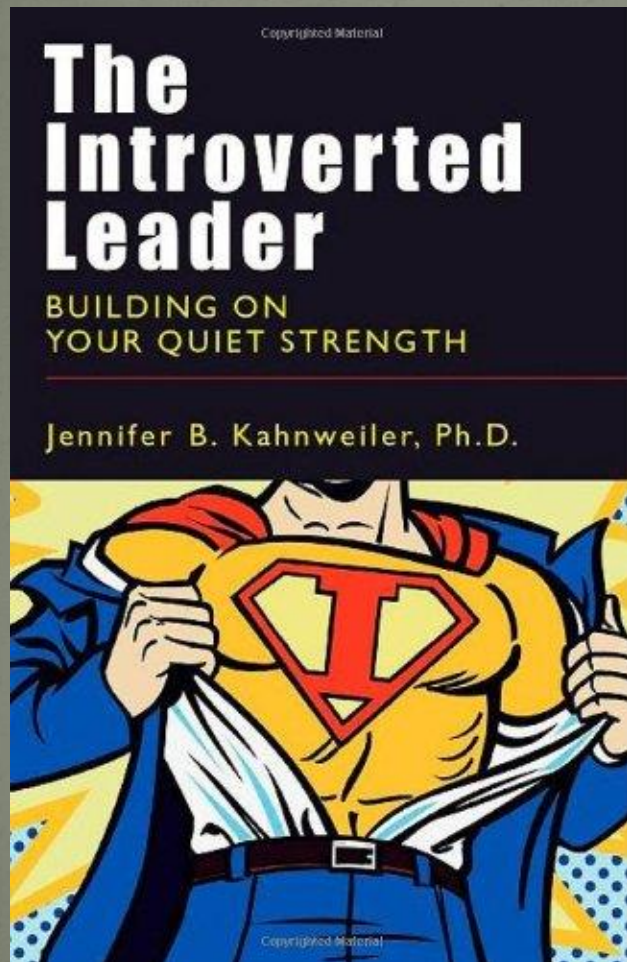


# Introvert Leaders

- “...Adam M. Grant, Francesca Gino, and David A. Hofmann conducted research that found some fallacy in the conventional wisdom, which is supported by years of academic research, that extroverts make the best leaders. They wrote in a **Harvard Business Review** article that their findings suggested that **extroverts and introverts were equally successful in leadership roles overall**, and that **introverts, in certain situations, actually make better bosses.**”

Read more: <http://www.businessinsider.com/why-introverts-can-be-the-best-leaders-2014-9>

# Introvert Leaders



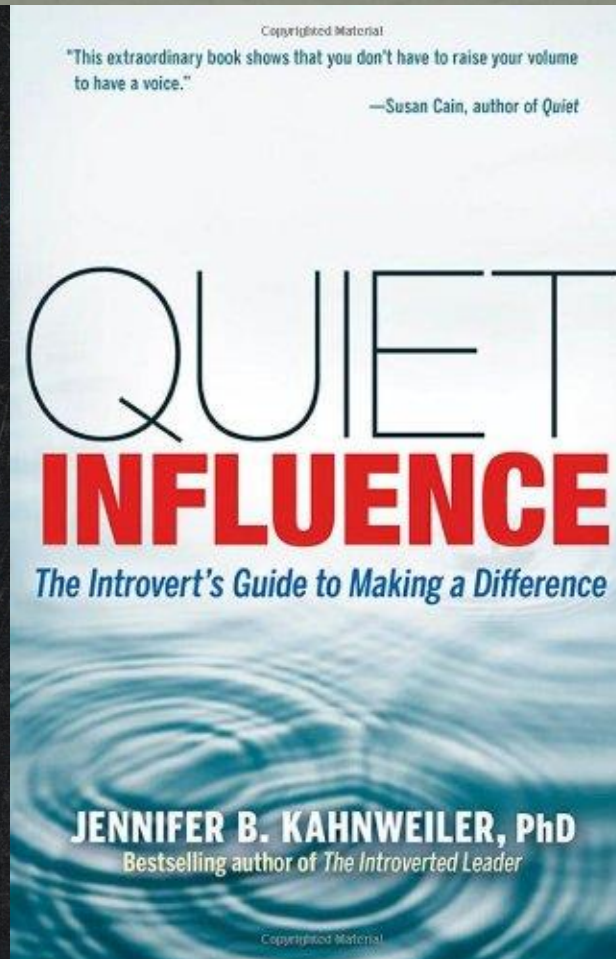
# Introvert Leaders

Introvert's Road to  
**LEADERSHIP**

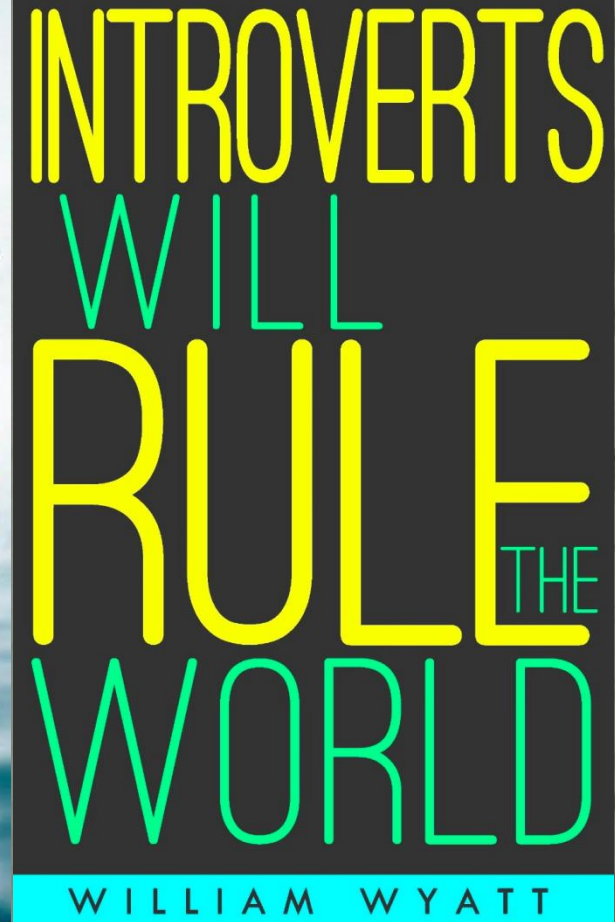
The Power of  
Being Quiet

Laurie Cain

Michigan State



WAVES Lab



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# Related Problem Areas

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- The interaction between users and content in multimedia social networks such as YouTube

# Importance of links in social graphs

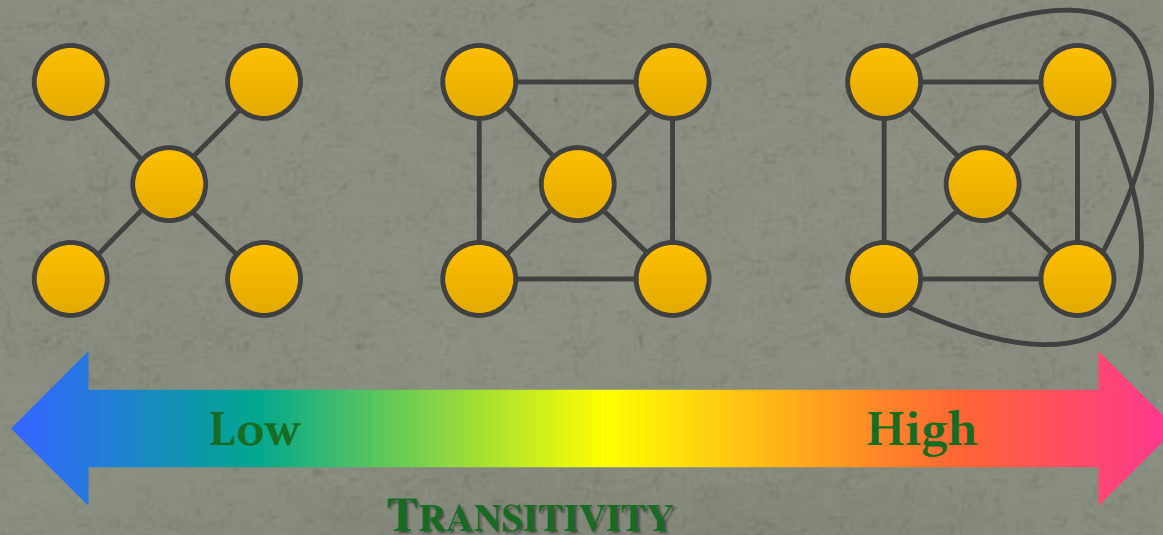
- Granovetter's seminal work "*The strength of weak ties*"
  - Importance of links that are perceived as "weak"
  - "Redundancy" of links that are perceived as "strong"
  - The paradox that exists between micro- and macro-level perception of social networks
- **Goal:** quantify the "strength of ties" in a topological sense that reflects social science theories, bridging micro- and macro-level views of social networks

Granovetter, "The strength of weak ties," in American Journal of Sociology, 1973  
Borgatti; Halgin, "On network theory," in Organization Science, 2011

# Network transitivity

(*Global clustering coefficient*)

- Transitivity is a global measure for how “cohesive” or “redundant” a network is
- The ratio of the number of triangles to the number of connected triples



# Transitivity matrix as a measure for the strength of individual links in a network

Transitivity Function

$$\tau(W) = \frac{\alpha}{\beta} = \frac{\text{trace}[W^3]}{\text{trace}[W^2H]}$$

Transitivity Gradient

$$\nabla_W \tau \triangleq \frac{\partial \tau}{\partial W}$$

Transitivity Matrix

$$T = \nabla_W \tau \odot W$$

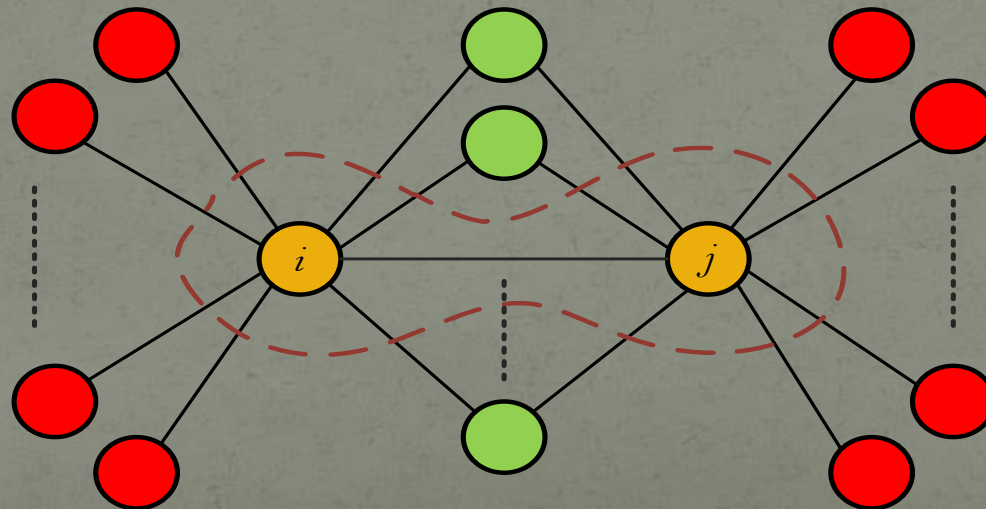
Aghagolzadeh, M.; and Radha, H.; "Transitivity Based Community Analysis and Detection", Proceedings of the IEEE Global Conference on Signal and Information Processing, December 2013 (Invited Paper)

# Transitivity Matrix: “Role” of individual links

$$T = \frac{3}{\beta} W^2 \odot W - \frac{\alpha}{\beta^2} (WH + HW) \odot W$$

*mutual neighbors*

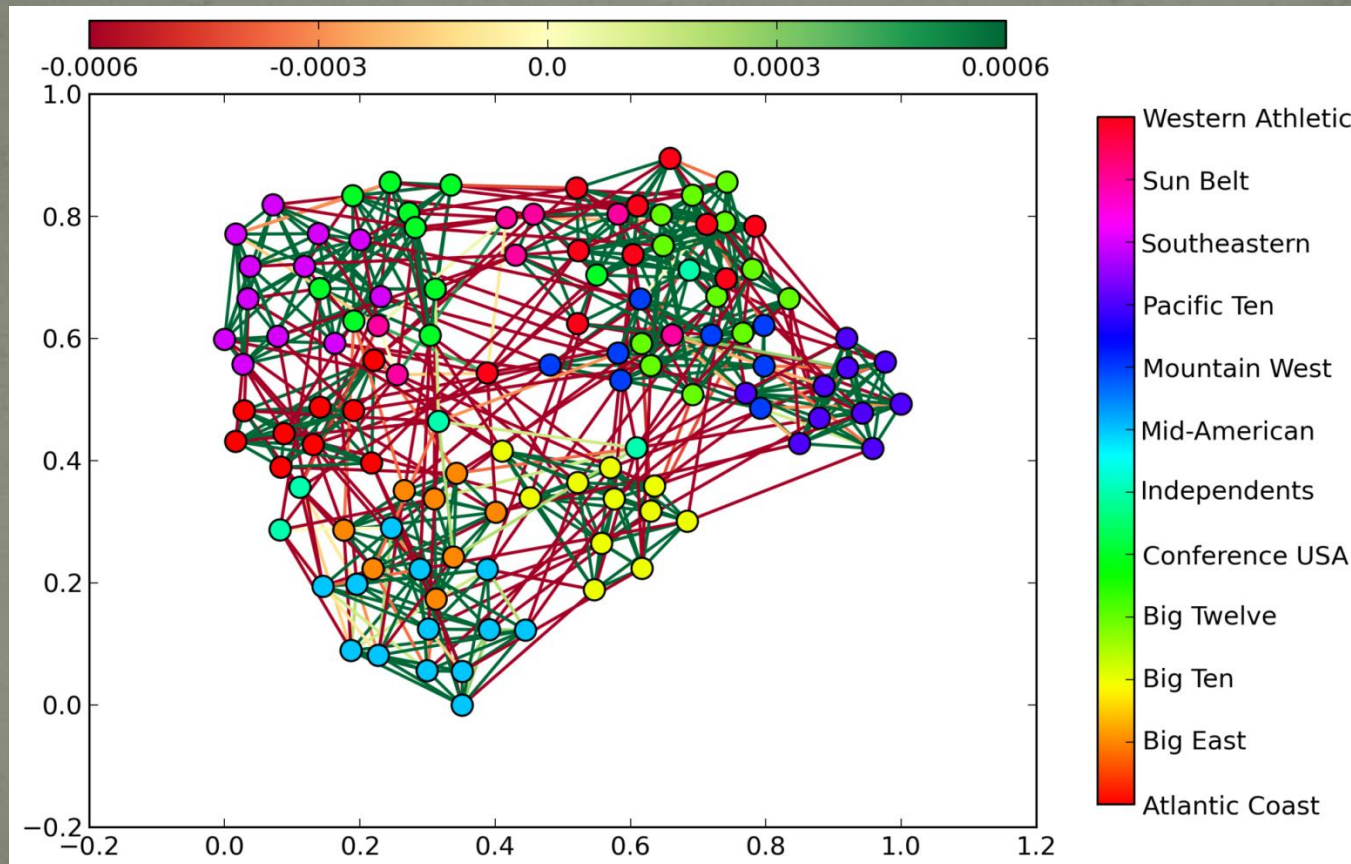
*combined degree*



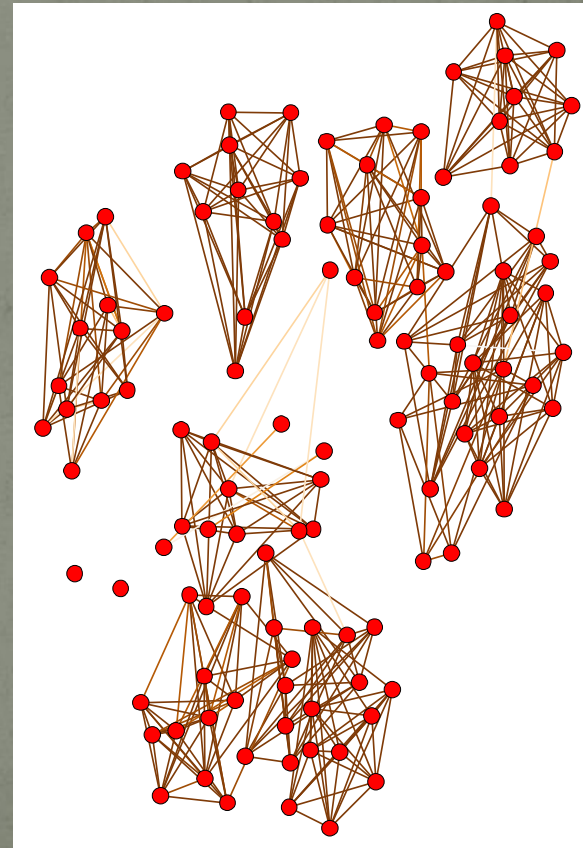
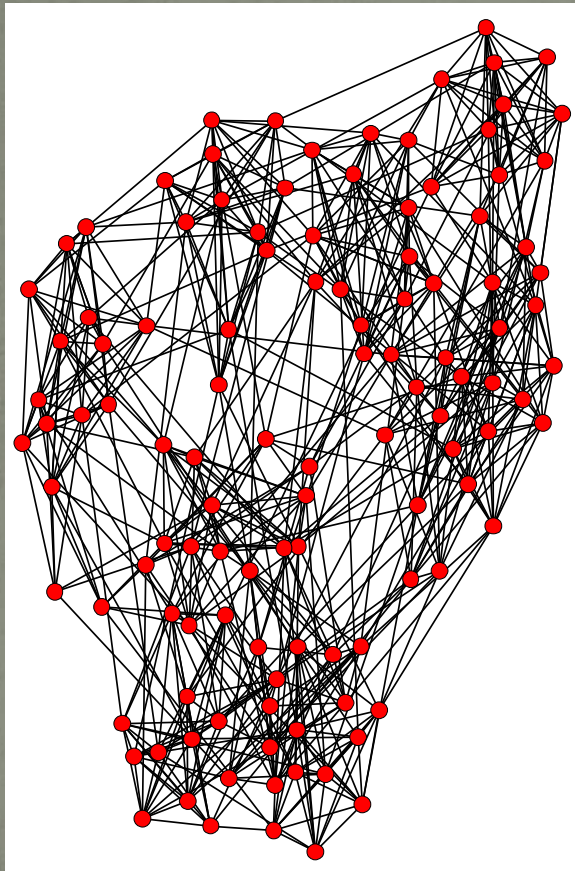


# Transitivity Matrix: “Role” of individual links

## Football network



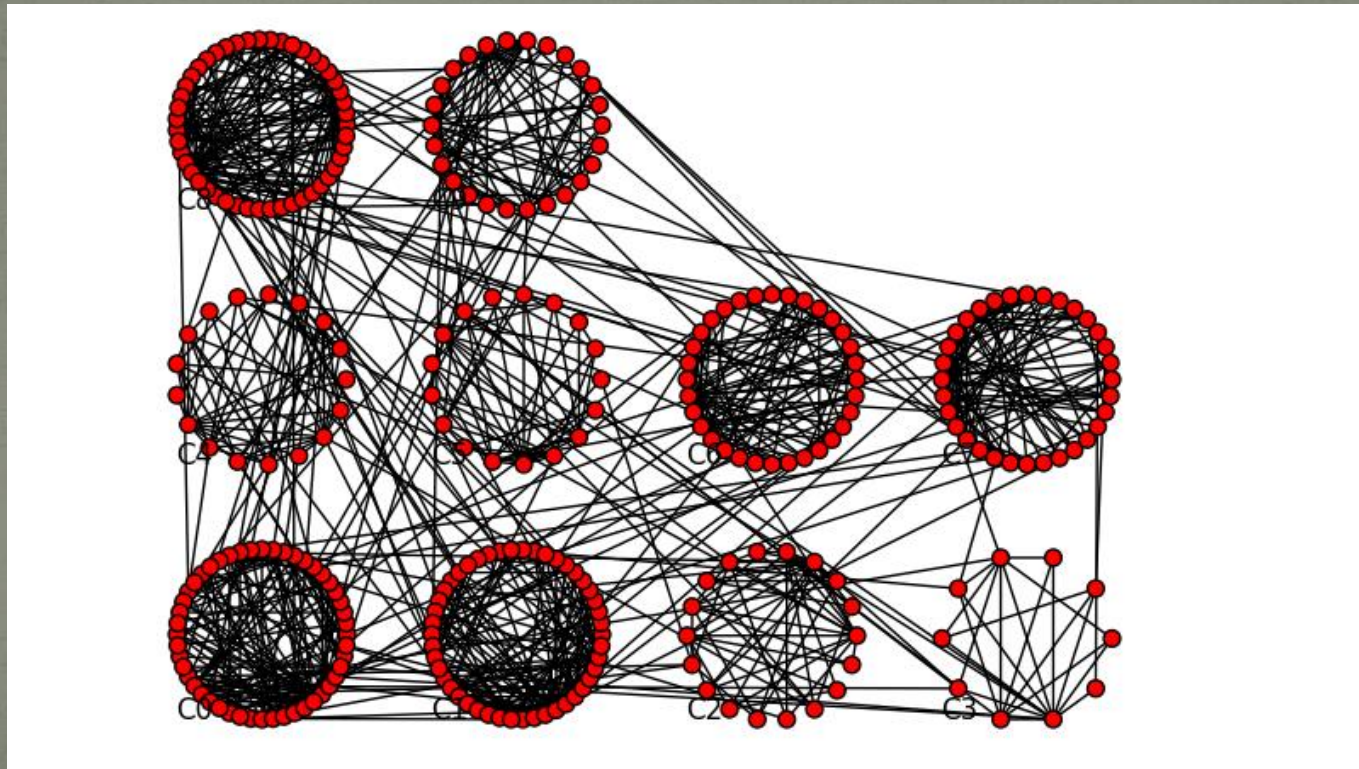
# Gradient of transitivity matrix and community detection





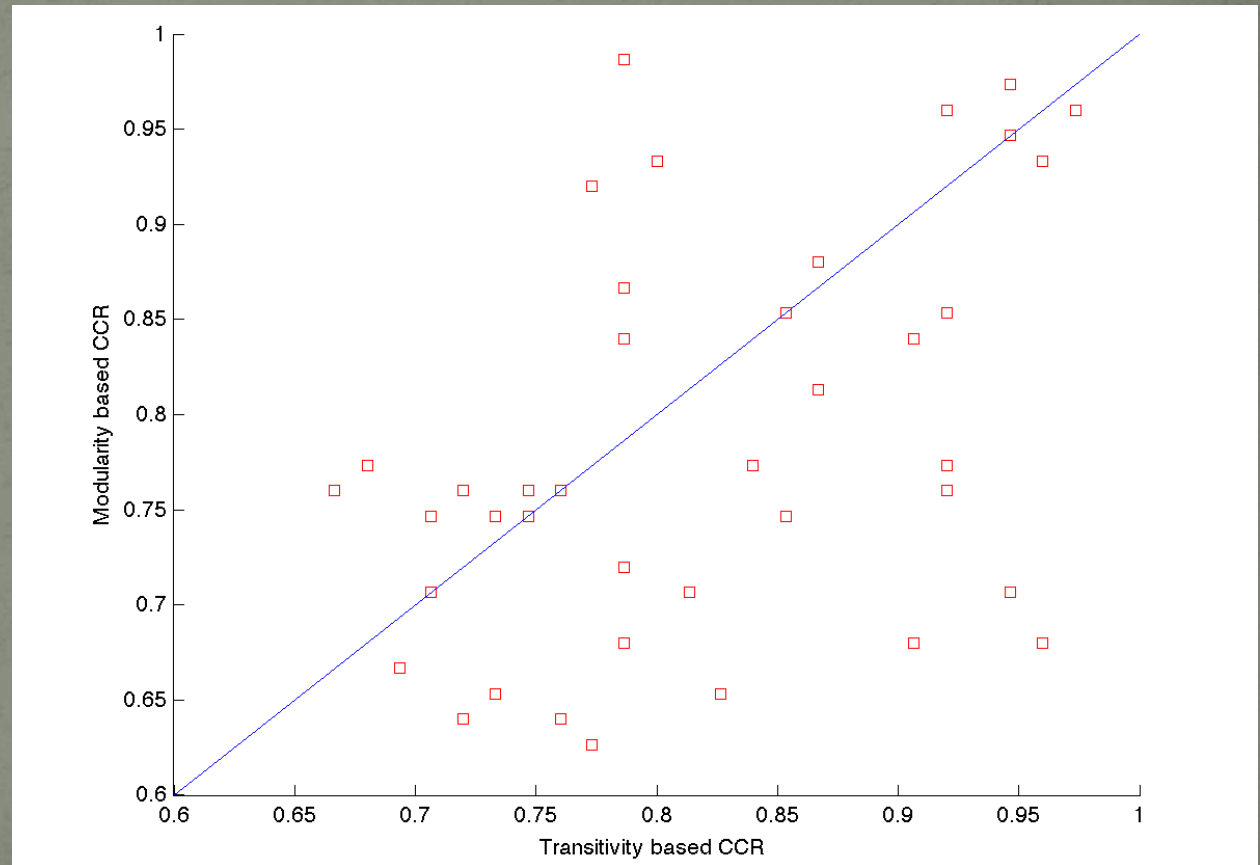
# Gradient of transitivity matrix and community detection

- Comparison with modularity based community detection



# Gradient of transitivity matrix and community detection

- Comparison with modularity based community detection



# Conclusions

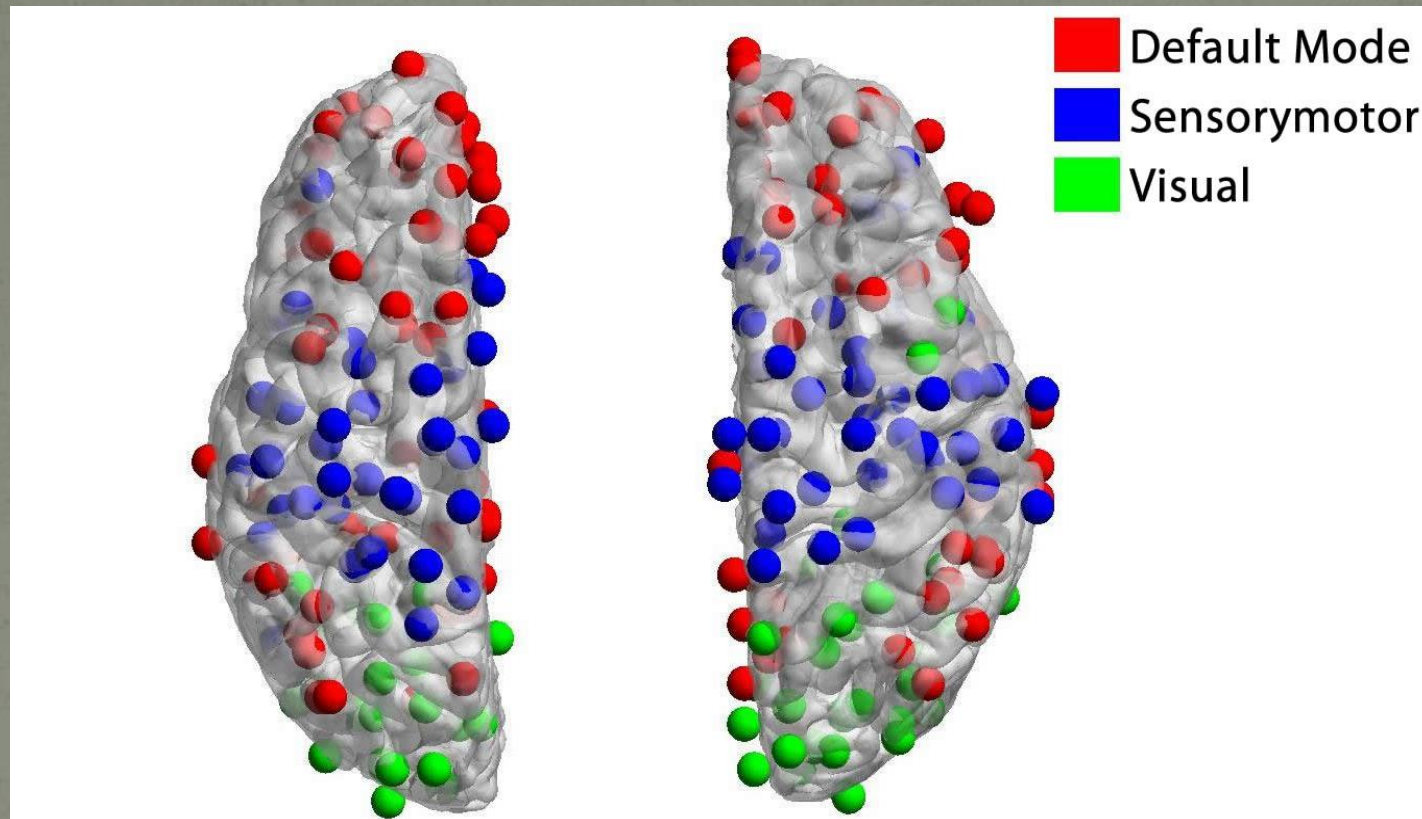
Multidimensional spectral analysis methods for massive graphs are more insightful than traditional approaches

New “Graph Transforms” can provide new insight into social networks, neural networks, sensor networks, etc.

Aspects of signal processing, graph theory, information theory and machine learning can be integrated to develop new analysis tools for massive network graphs

# Role of edges in brain networks

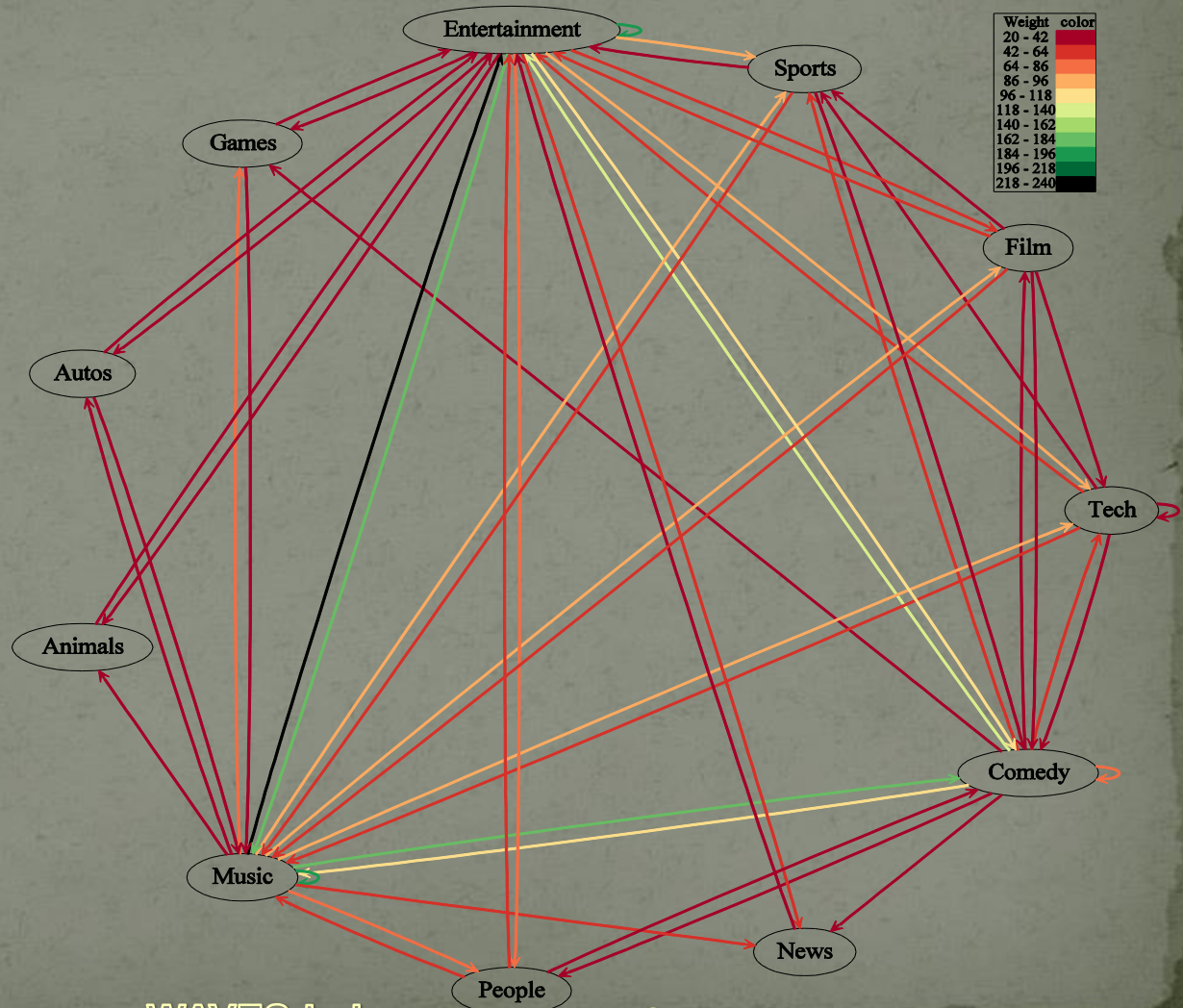
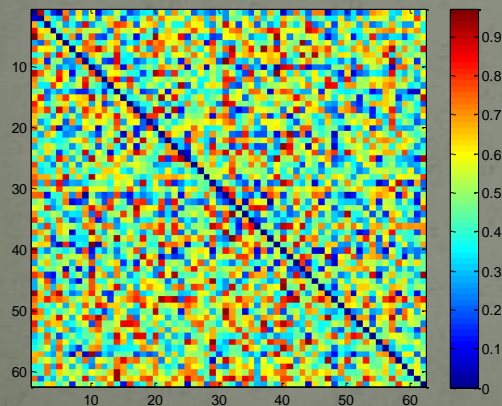
(McGovern Institute for Brain Research, MIT)



Yu-Teng Chang, Dimitrios Pantazis, McGovern Institute for Brain Research, Massachusetts Institute of Technology  
MODULARITY GRADIENTS: MEASURING THE CONTRIBUTION OF EDGES TO THE COMMUNITY STRUCTURE OF A  
BRAIN NETWORK; 2013 IEEE 10th International Symposium on Biomedical Imaging

# On-going work: Youtube viewership analysis...

- Analysis of the flow of viewership using causality





# More details can be found in....

- Aghagolzadeh, M.; and Radha, H.; “Transitivity Based Community Analysis and Detection,” Proceedings of the IEEE Global Conference on Signal and Information Processing, December 2013 **(Invited Paper)**
- Aghagolzadeh, M.; and Radha, H.; “Denoising of Network Graphs using Topology Diffusion,” Proceedings of Asilomar, November 2014 **(Invited Paper)**
- Ilyas, M.U.; Shafiq, M.Z.; Liu, A.X.; Radha, H., "A Distributed Algorithm for Identifying Information Hubs in Social Networks," *Selected Areas in Communications, IEEE Journal on (JSAC)*, vol.31, no.9, pp.629,640, September 2013.
- Shafiq, M.Z.; Ilyas, M.U.; Liu, A.X.; Radha, H., "Identifying Leaders and Followers in Online Social Networks," *Selected Areas in Communications, IEEE Journal on (JSAC)*, vol.31, no.9, pp.618,628, September 2013.
- Aghagolzadeh, M.; Barjasteh, I.; Radha, H.; , "Transitivity matrix of social network graphs," *Statistical Signal Processing Workshop (SSP), 2012 IEEE* , vol., no., pp.145-148, 5-8 Aug. 2012
- Muhammad U. Ilyas and Hayder Radha , "Identifying Influential Nodes in Online Social Networks Using Principal Component Centrality," *Proceedings of the IEEE International Conference on Communications (ICC'11)*, Kyoto, Japan, June 5-9, 2011.
- Muhammad U. Ilyas, M. Zubair Shafiq, Alex X. Liu, and Hayder Radha , "A Distributed and Privacy-Preserving Algorithm for Identifying Information Hubs in Social Networks," *Proceedings of the 30th IEEE International Conference on Computer Communications (INFOCOM'11)*, April 10 - 15, 2011.

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