The Role of Microeconomic Theory in Networked Systems' Performance

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in collaboration with

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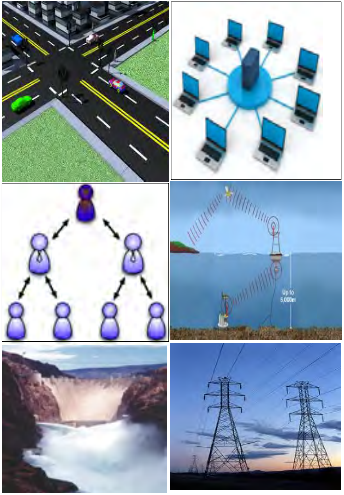
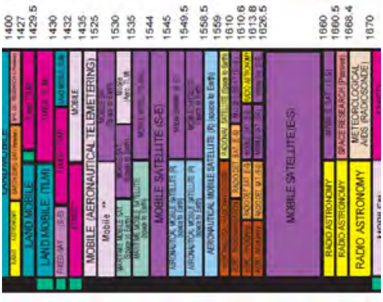
Decentralized Systems

- Energy Markets
- Communication Networks
- Social Networks
- Auctions
- Transportation Systems
- Networked Control Systems
- Environmental Monitoring Systems

⋮



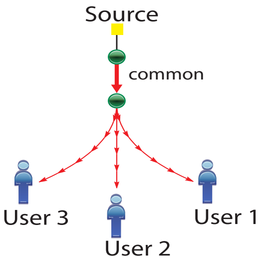
Radio Spectrum Allocation in Wireless Networks



- Radio waves are what allow cell phones to communicate.
- Radio waves travel across different frequencies of radio spectrum.
- In the U.S., every business or individual who wants to broadcast using radio waves must acquire a license from the FCC.
- If several agents use the same frequency band, they experience interference that potentially reduces their benefits.

  -what is an efficient way the FCC can use to allocate radio spectrum to strategic agents?

Bandwidth Allocation in Wired Networks



- Agents **share** a link with a finite bandwidth capacity.
- Agents have different and **Private** values/utilities.

  -*what is the optimal (maximizing social welfare) way to allocate the finite rate/bandwidth to a set of agents?*

Economic Systems: Auctions

- Buyers in auctions make bidding decisions based on their private valuation of the object being sold.

  - *what is the optimal auction that allocates the object to the buyer whose value is the highest (maximizing revenue)?*

Energy Systems: Energy Procurement from a Strategic Energy Generator

- Strategic Buyer

  - Utility Maximizer

- Strategic Seller

  - Conventional and renewable energy resources

  - Private generation technology

  - *How should the buyer negotiate and sign a contract for energy procurement with the seller?*

What are the **common** features of these problems?

Key features in decentralized resource allocation problems

- Many **agents** have to **share** the system's **limited resources**.

- Each **agent** may have **different information** about the system, and agents' decisions/actions affect the other agents' information.

- **Agents** may behave **strategically**. They exchange information with one another and determine communication and decision strategies that lead to the objective of the problem (network objective).

Mechanism Design Theory:

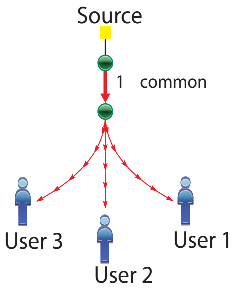
Formal treatment of decentralized resource allocation problems with **strategic** agents.

What is a **resource allocation problem**?
(centralized problem)

A resource allocation problem

Bandwidth allocation in wired networks:

- Environment space: U_1, U_2, U_3, and the fixed network
- Allocation Space: Bandwidth (x_1, x_2, x_3)
- Objective: Maximizing social welfare



The centralized problem:

$$
\begin{aligned}
\max_{(x_1,x_2,x_3)} \quad & U_1(x_1) + U_2(x_2) + U_3(x_3) \\
\text{s.t.} \quad & x_1 + x_2 + x_3 \leqslant 1 \\
& x_1 \geqslant 0 \\
& x_2 \geqslant 0 \\
& x_3 \geqslant 0
\end{aligned}
$$

A resource allocation problem

Bandwidth allocation in wired networks:

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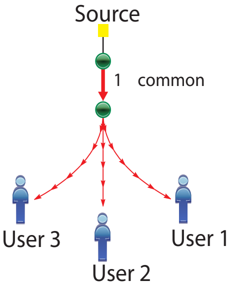


The centralized problem:

$$\begin{aligned} \max_{(x_1,x_3,x_3)} \quad & U_1(x_1) + U_2(x_2) + U_3(x_3) \\ \text{s.t.} \quad & x_1 + x_2 + x_3 \leqslant 1 \\ & x_1 \geqslant 0 \\ & x_2 \geqslant 0 \\ & x_3 \geqslant 0 \end{aligned}$$

Utilities are **private** information

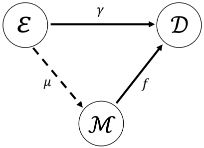
Agents are **strategic**

What is **Implementation Theory**?

Mechanism design

Implementation theory deals with Strategic agents.

- **Mechanism/game form**: $(\mathcal{M}, f)$



- Message/Strategy space $\mathcal{M}$: Set of messages agents can communicate to other agents.

- Outcome function f: Determines resource allocations at each message profile.

→ Game $(\mathcal{M}, f, \{u_i\})$ is induced by game form $(\mathcal{M}, f)$:
Players – network agents; Strategy set – $\mathcal{M}$; Players' utilities – $\{u_i(f(\boldsymbol{m}))\}$.

→ Equilibrium/solution concept: Nash equilibrium, Bayesian Nash equilibrium, etc. *Appropriate solution concept determined by the information structure of the game.*

Nash equilibrium (NE):

- A message profile $\boldsymbol{m}^*_{\mathcal{N}} := (\boldsymbol{m}^*_1, \boldsymbol{m}^*_2, \ldots, \boldsymbol{m}^*_N)$ with property

$$u_i\big(f_i(\boldsymbol{m}^*_{\mathcal{N}})\big) \geqslant u_i\big(f_i(\boldsymbol{m}_i, \boldsymbol{m}^*_{\mathcal{N}/i})\big) \quad \forall\, \boldsymbol{m}_i \in \mathcal{M}_i,\ \forall\, \boldsymbol{i} \in \mathcal{N}.$$

⋆

Desirable properties of a **Mechanism**

Desirable properties of a mechanism/game form:

(I) Implementation in Nash equilibria: A game form $(\mathcal{M}, f)$ "implements the goal correspondence γ in Nash equilibria" if, for all problem environments,

Set of allocations at **All Nash equilibria** $\subset$ Set of optimal centralized allocations

$\equiv$ **ALL** the **Nash equilibria** of the game induced by the game form are **efficient**.

Desirable properties of a mechanism/game form:

(II) Individual rationality: A game form $(\mathcal{M}, f)$ is individually rational if, for all agents,

Utility at all Nash equilibria $\geqslant$ Utility before/without participating in the allocation process specified by the game form

$\equiv$ All the **agents** **VOLUNTARILY** **participate** in the game induced by the game form.

Desirable properties of a mechanism/game form:

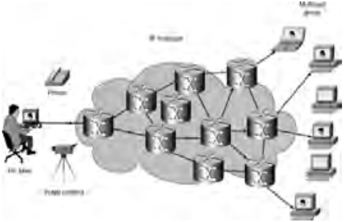
(III) Budget balance: A game form $(\mathcal{M}, f)$ is budget balanced if,

Net money transfer in the system = 0

$\equiv$ **Sum** of the **taxes** is *always* equal to **zero**.

Multi-rate Multicast service provisioning problem with **strategic** agents

Multi-rate Multicast **Technology**
History



- Multi-rate Multicast technology is used as a protocol in wired networks.
- Initiated in 1990, (called **MBONE**), by USC, MIT and the Lawrence Berkeley National Lab.
- **Large-scale widely used communication infrastructure**

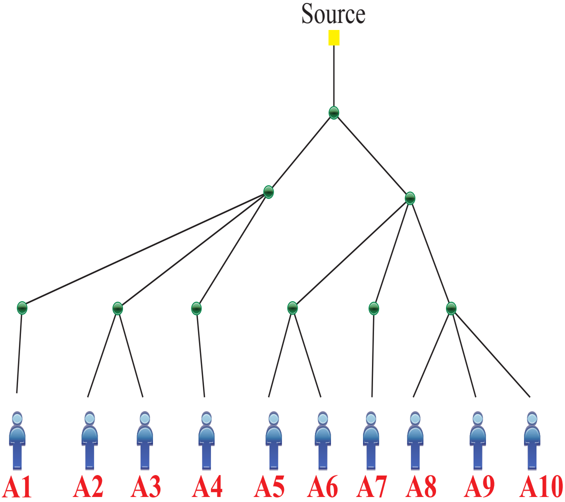
Literature Survey

- See A. Kakhbod, D. Teneketzis, "An Efficient Game Form for Multi-rate Multicast Service Provisioning" *IEEE Journal on Selected Areas in Communication, Special Issue on the Economics of Communication Networks and Systems*, Vol. 30, No. 10, December 2012, pp. 2093-2104

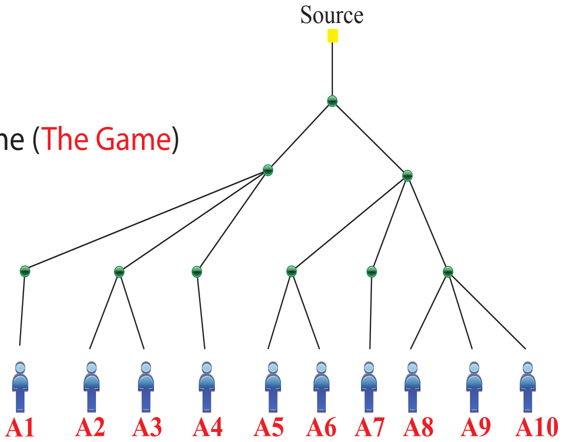
- All previous literature assumes non-strategic users.

Multi-rate Multicast **Technology**

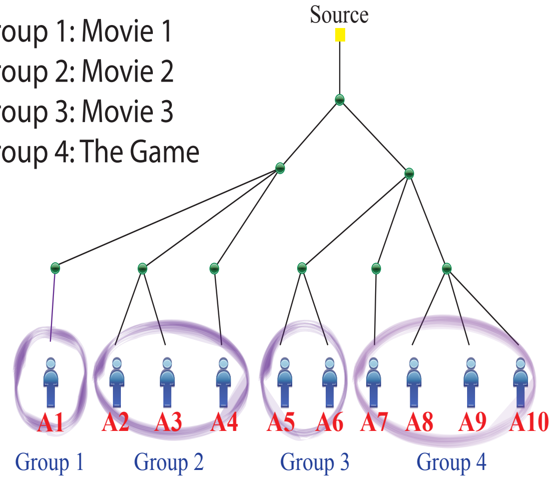
The main feature



4 types of Programs:
Movie 1
Movie 2
Movie 3
The super bowl game (The Game)



Group 1: Movie 1
Group 2: Movie 2
Group 3: Movie 3
Group 4: The Game



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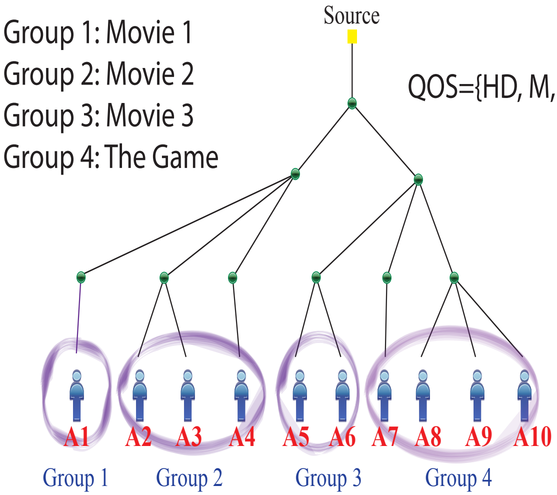

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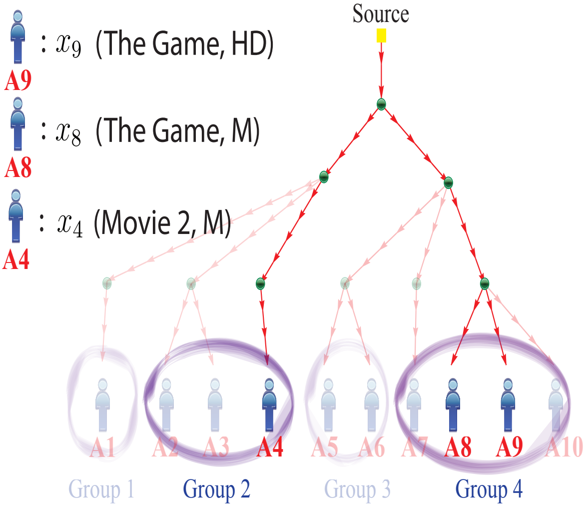

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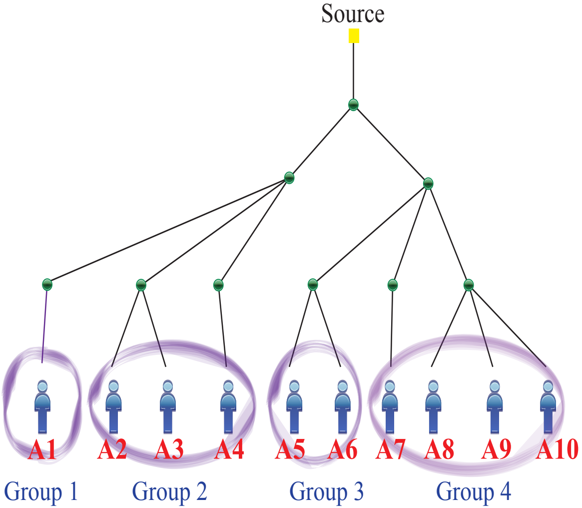

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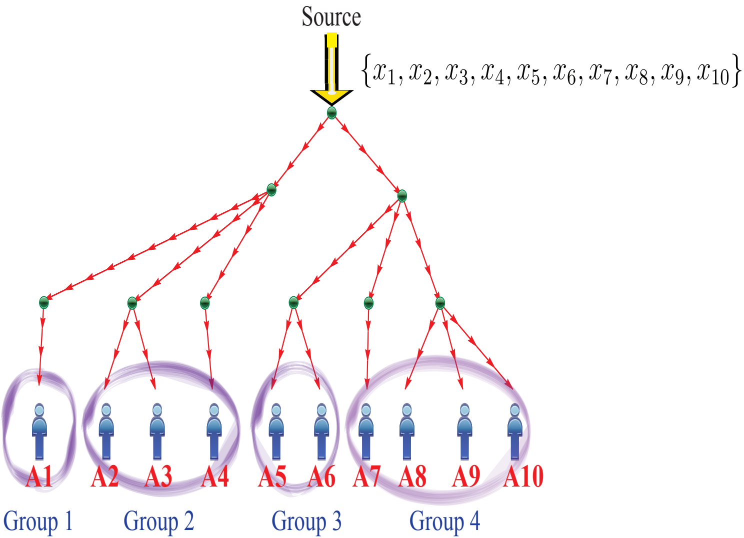

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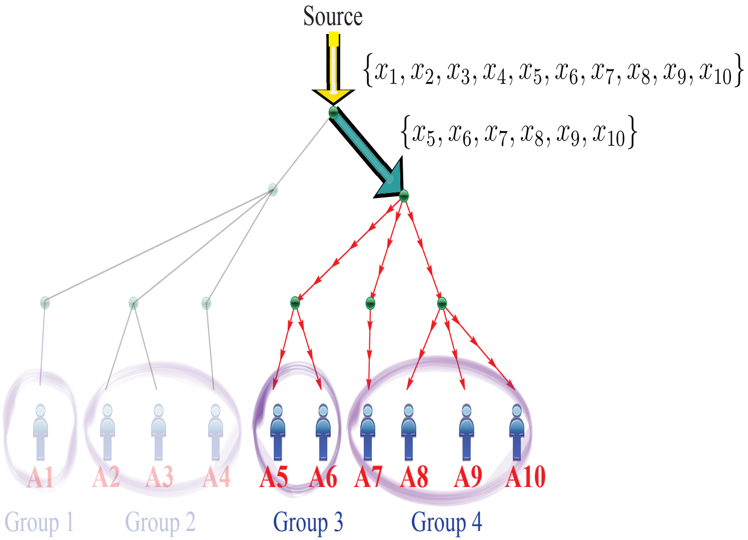

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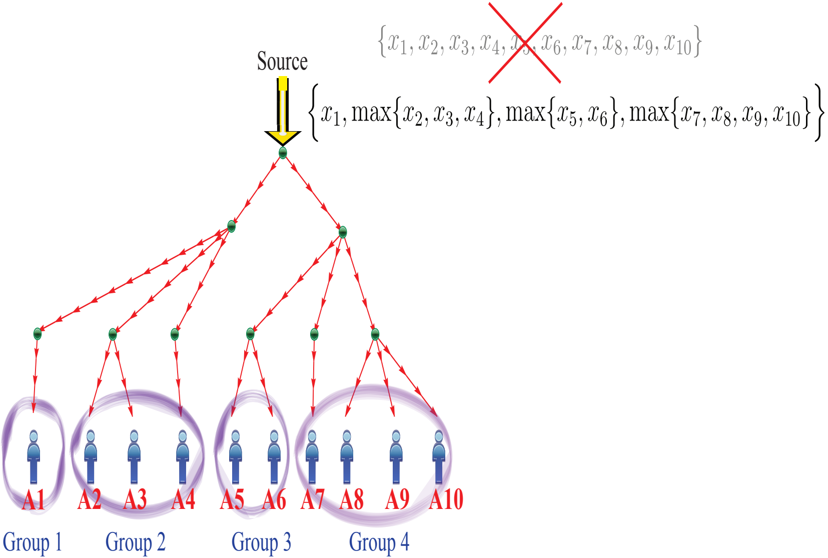
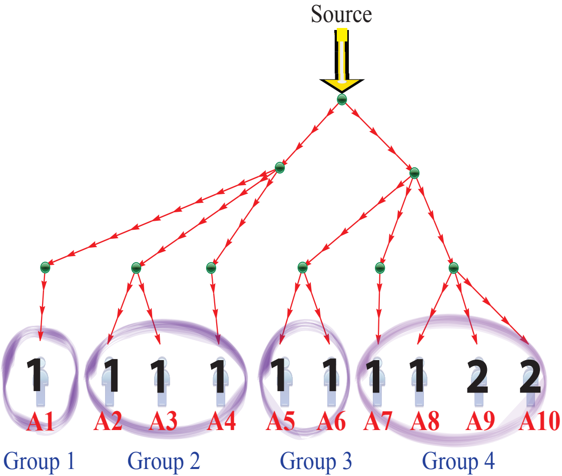
The main feature



Multi-rate Multicast **Technology**

The main feature

(Movie i, M), (The Game, M) $\rightarrow$ 1
(The Game, HD) $\rightarrow$ 2
A_1: (Movie1, M), A_2, A_3, A_4: (Movie 2, M), A_5, A_6: (Movie 3, M)
A_7, A_8: (The Game, M), A_9, A_{10}: (The Game, HD),



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The main feature

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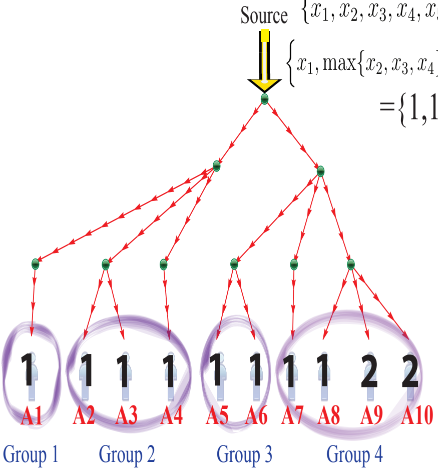


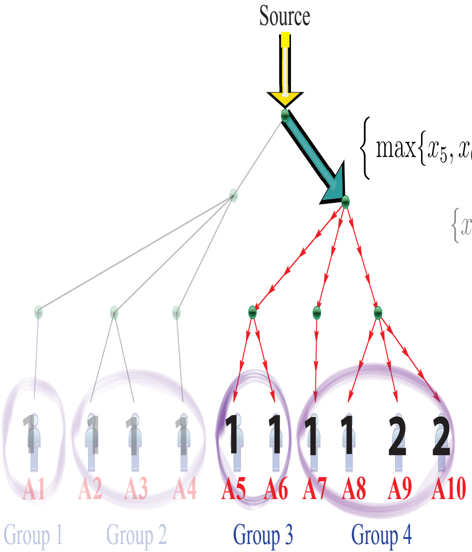
$$\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\} = \{1,1,...,1,2,2\} \rightarrow 12$$

$$\{x_1, \max\{x_2, x_3, x_4\}, \max\{x_5, x_6\}, \max\{x_7, x_8, x_9, x_{10}\}\} = \{1,1,1,2\} \rightarrow 5$$

12-5=7 (SAVED)

Multi-rate Multicast **Technology**

The main feature



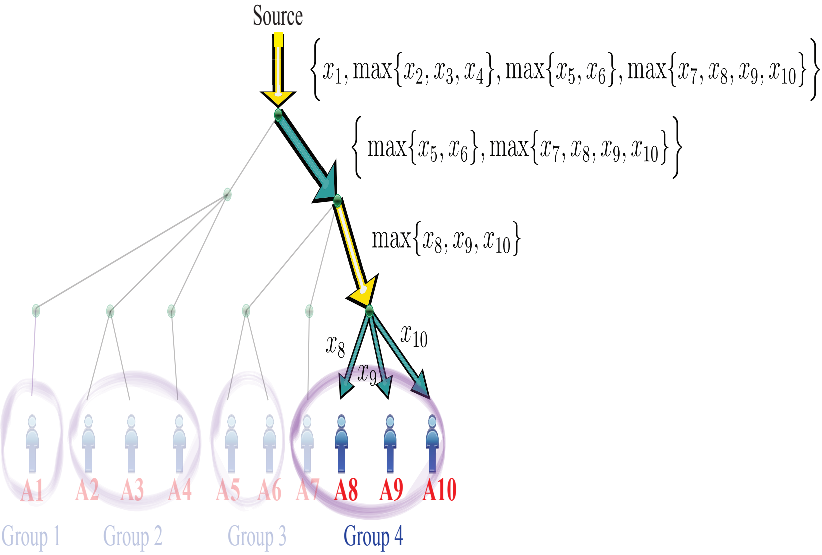
$\left\{ \max\{x_5, x_6\}, \max\{x_7, x_8, x_9, x_{10}\} \right\}$={1,2} → **3**

$\{x_5, x_6, x_7, x_8, x_9, x_{10}\}$={1,1,1,1,2,2} → **8**

8-3=5 (SAVED)

Multi-rate Multicast **Technology**

The main feature



What is the **Centralized** problem?

Multi-rate Multicast Service Provisioning Problem

The Centralized Problem (P_C)

$$\max_{(x_1, x_2, \cdots, x_{10})} \quad U_1(x_1) + U_2(x_2) + \cdots + U_{10}(x_{10})$$

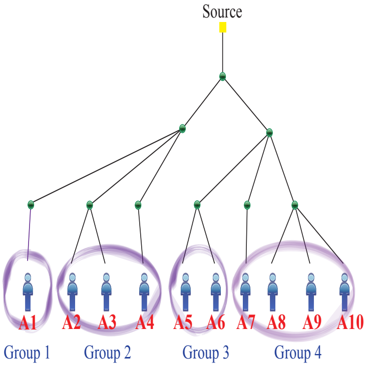
s.t. **Capacity** Constraint at **each** link is satisfied

$$x_1 \geqslant 0$$

$$x_2 \geqslant 0$$

$$\vdots$$

$$x_{10} \geqslant 0$$



Multi-rate Multicast Service Provisioning Problem

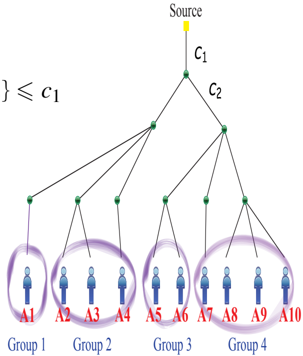
The Centralized Problem (P_C)

$$\max_{(x_1,x_2,\cdots,x_{10})} \quad U_1(x_1) + U_2(x_2) + \cdots + U_{10}(x_{10})$$

$$\text{s.t.} \quad x_1 + \max\{x_2, x_3, x_4\} + \max\{x_5, x_6\} + \max\{x_7, x_8, x_9, x_{10}\} \leqslant c_1$$

$$\max\{x_5, x_6\} + \max\{x_7, x_8, x_9, x_{10}\} \leqslant c_2$$

$$\vdots$$



Problem ($\boldsymbol{P_C}$)

$$\begin{aligned}
\max_{(\boldsymbol{x}_{\mathcal{N}})} \quad & \sum_{G_i \in \mathcal{N}} \sum_{(j,G_i) \in G_i} U_{(j,G_i)}(x_{(j,G_i)}) \\
\text{s.t.} \quad & \sum_{G_i \in Q_l} \max_{(j,G_i) \in G_i(l)} x_{(j,G_i)} \leqslant c_l \qquad \forall\, l \in \mathbf{L} \\
& x_{(j,G_i)} \geqslant 0 \qquad \forall\, (j, G_i)
\end{aligned}$$

Key **difficulties** in obtaining a **centralized optimal solution**!

(1) Information is Decentralized.

→ Every agent's utility function is its **private** information.
→ Utility function of agent i, $V_i(x_i, t_i) = u_i(x_i) - t_i$
$t_i \rightarrow$ tax

(2) Agents are Strategic.

Above difficulties imply that we must solve a *decentralized optimization problem* with *strategic* agents.

Decentralized optimization problem with strategic agents: **Problem formulation**

(I1) Each agent knows *only* his own utility. ($V_i(x_i, t_i) = u_i(x_i) - t_i$)

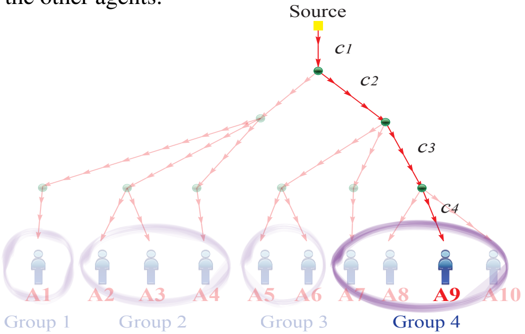
(I2) Each agent behaves *strategically*. The agent's objective is to maximize his own utility function.

(I3) The network operator knows the topology and resources of the network.

(I4) The network operator receives requests for service and announces to each agent

- The group to which the agent belongs.
- The set of links that form agent's route.
- The capacity of each link in his route.

(I5) Each strategic agent competes for resources (bandwidth) at each link of his route with the other agents.



Objective

Develope a **game form** that

(1) is individually rational.

(2) results in budget balance.

(3) implements in Nash equilibria the optimal solution of Problem (P_C)
ALL the NE are EFFICIENT.

Problem (P_C)

$$\max_{(\boldsymbol{x}_{\mathcal{N}})} \quad \sum_{G_i \in \mathcal{N}} \sum_{(j,G_i) \in G_i} U_{(j,G_i)}(x_{(j,G_i)})$$

$$\text{s.t.} \quad \sum_{G_i \in Q_l} \max_{(j,G_i) \in G_i(l)} x_{(j,G_i)} \leqslant c_l \qquad \forall\, l \in \mathbf{L}$$

$$x_{(j,G_i)} \geqslant 0 \qquad \forall\, (j, G_i)$$

Contribution:

Developed a **game form** that

(1) **is individually rational.**

(2) **results in budget balance at equilibrium.**

(3) **implements in Nash equilibria the optimal solution of Problem (P_C)**
ALL the NE are EFFICIENT.

Problem (P_C)

$$\max_{(\boldsymbol{x}_{\mathcal{N}})} \quad \sum_{G_i \in \mathcal{N}} \sum_{(j,G_i) \in G_i} U_{(j,G_i)}(x_{(j,G_i)})$$

$$\text{s.t.} \quad \sum_{G_i \in Q_l} \max_{(j,G_i) \in G_i(l)} x_{(j,G_i)} \leqslant c_l \qquad \forall\, l \in \mathbf{L}$$

$$x_{(j,G_i)} \geqslant 0 \qquad \forall\, (j, G_i)$$

What are the main features of the **game form/mechanism**?

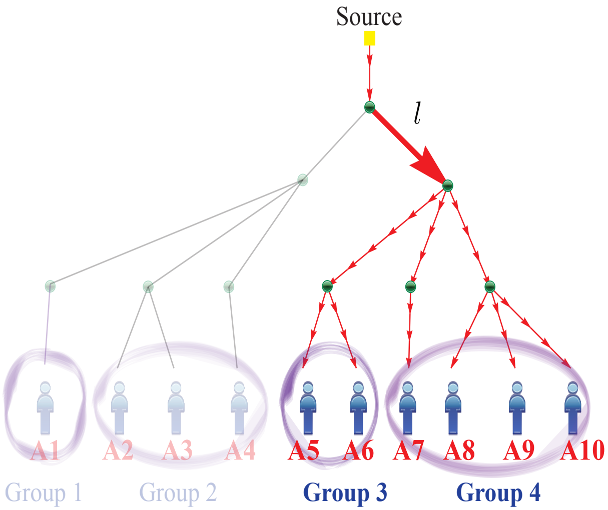
Multi-rate Multicast service provisioning with strategic agents is the combination of a Market problem and a Public good problem

- The resource allocation among groups is a market problem.
- The resource allocation among the agents of the same group is a public good problem.

Multi-rate Multicast Service Provisioning with **Strategic agents**

Key features of the Game form

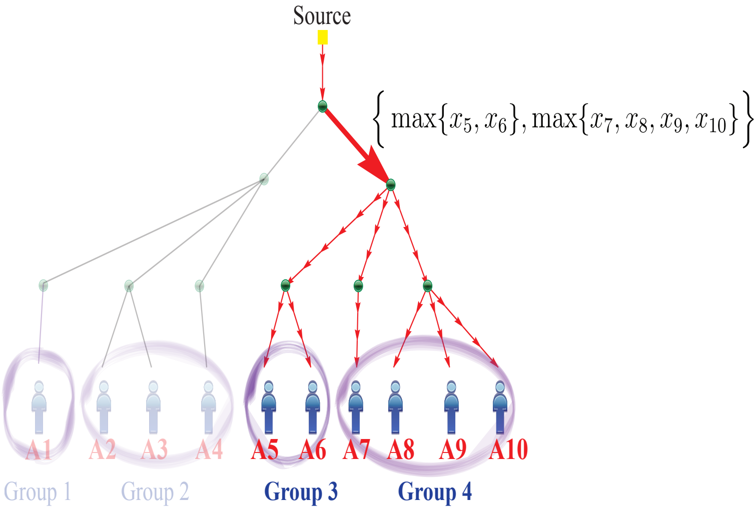
The Market component:



Multi-rate Multicast Service Provisioning with **Strategic agents**

Key features of the Game form

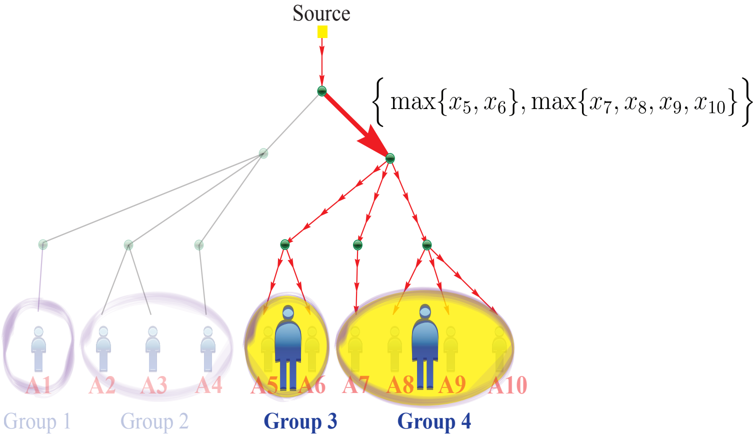
The **Market** component:



Multi-rate Multicast Service Provisioning with **Strategic agents**

Key features of the Game form

The **Market** component:

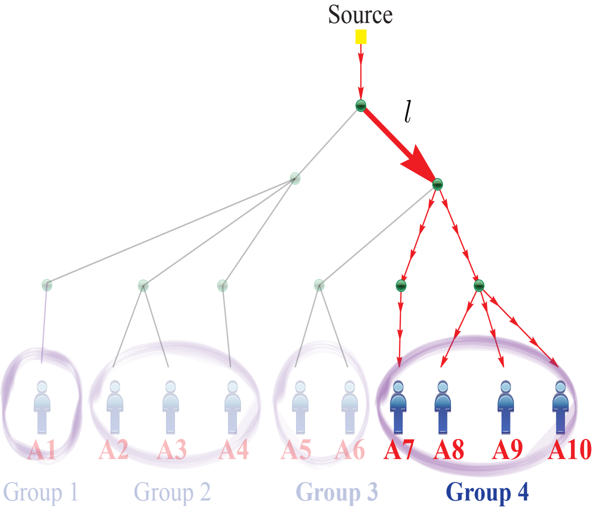


→ **A. Kakhbod** and D. Teneketzis, "An efficient game form for unicast service provisioning problem," **IEEE Transaction on Automatic Control (TAC)**, **vol 57**, no. 2, pp. 392 - 404, 2012.

Multi-rate Multicast Service Provisioning with **Strategic agents**

Key features of the Game form

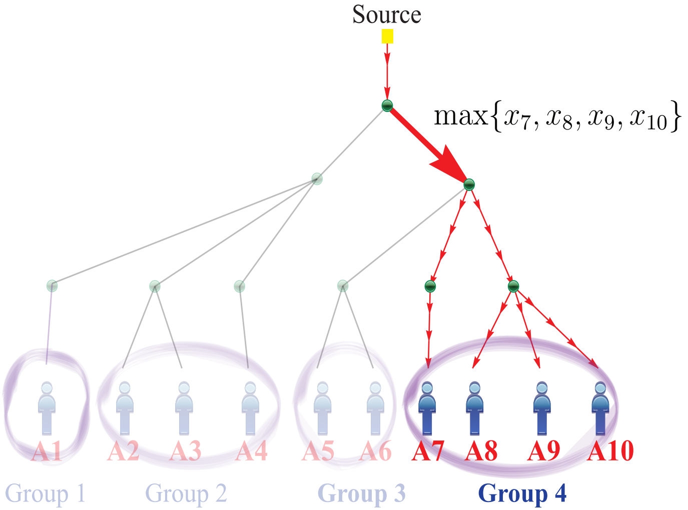
The Public good component:



Multi-rate Multicast Service Provisioning with **Strategic agents**

Key features of the Game form

The Public good component:



Specification of

M :=**Message Space** and f :=**Outcome function**

Specification of the game form

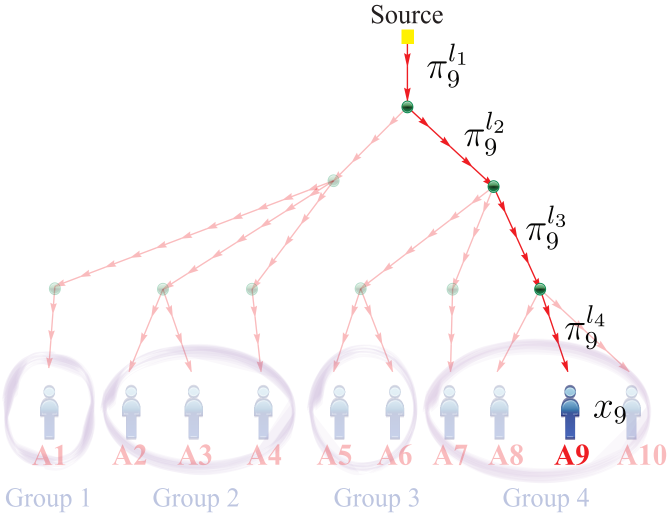
Message Space: $M := \bigotimes_{G_i \in \mathcal{N}} \bigotimes_{(j,G_i) \in G_i} M_{(j,G_i)}$

$$\mathbf{m}_{(j,G_i)} = \left(x_{(j,G_i)}, \pi_{(j,G_i)}^{l_{j_1}}, \pi_{(j,G_i)}^{l_{j_2}}, \cdots, \pi_{(j,G_i)}^{l_{j_{|R_{(j,G_i)}|}}} \right)$$

- $x_{(j,G_i)}$:= bandwidth/rate agent (j, G_i) requests
- $\pi_{(j,G_i)}^{l_{j_k}}$:= price per unit of bandwidth agent (j, G_i) is willing to pay at link l_{j_k} of his route.

Specification of the game form

Message Space: $M := \bigotimes_{G_i \in \mathcal{N}} \bigotimes_{(j,G_i) \in G_i} M_{(j,G_i)}$



Specification of the game form
Outcome function: f

For any message profile $\mathbf{m}, \mathbf{m} \in M$,

$$f(\mathbf{m}) = \left[(\text{bandwidth}, \ \mathbf{tax})_{(j,G_i)}, \cdots\right]$$

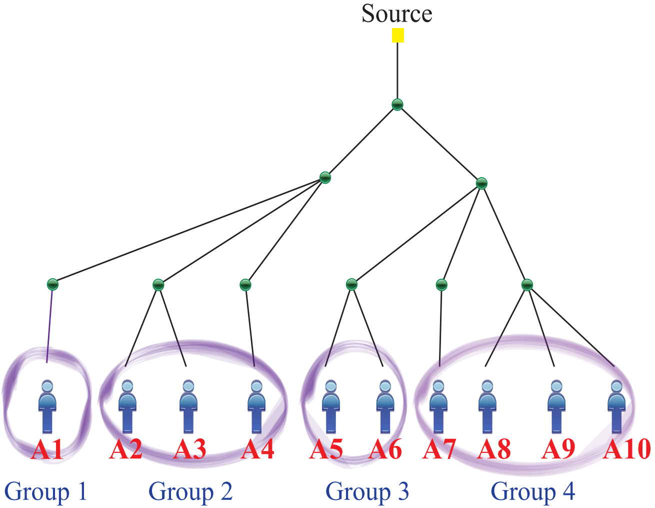
bandwidth and **tax** for each agent

For each agent

The tax (subsidy) is defined at each link of its route based on the number of groups using the link.

Specification of the game form

Outcome function: f



Specification of the game form

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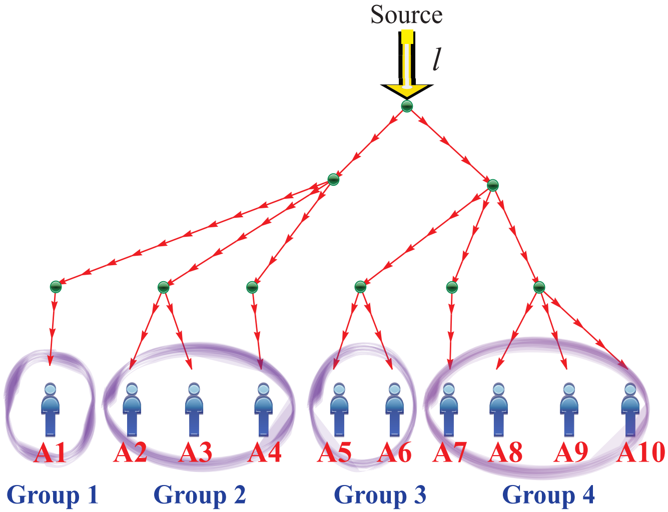
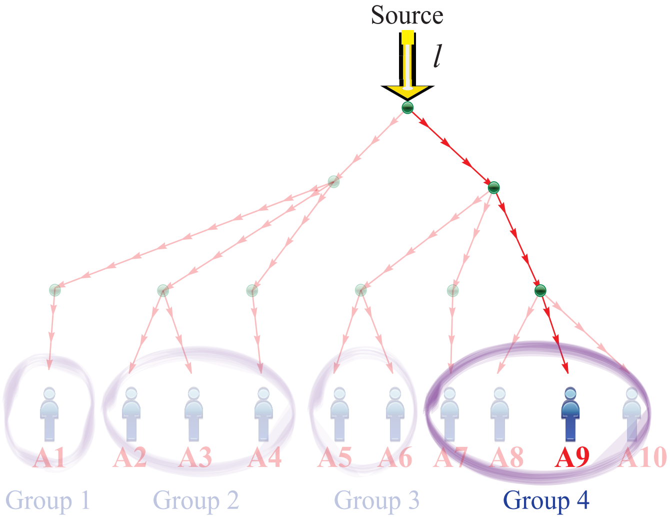


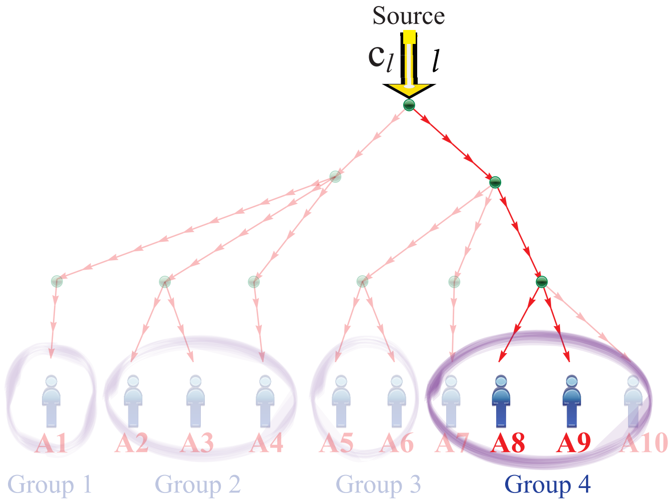
Specification of the game form

Outcome function: f



Outcome function: f

A_8 and **A_9** request the **maximum** bandwidth in **G_4** at link l



Interpretation of the tax function

Public good terms, Market terms

$$t^l_{A9} = \left[\Delta_2^{A9}(l) + \Delta_3^{A9}(l) + \Delta_4^{A9}(l)\right] \times \Delta_1^{A9}(l).$$

$$\Delta_1^{A9}(l) \quad := \quad \mathbb{I}\{x_9 \text{ is the the maximum request in } G_4 \text{ at link } l\}$$

$$\Delta_2^{A9}(l) \quad := \quad \pi^l_8 \; x_9$$

$$\Delta_3^{A9}(l) \quad := \quad \frac{\left(P_{G_4(l)} - P_{-G_4(l)} - \eta^l_+\right)^2}{2} - P_{-G_4(l)}\left(P_{G_4(l)} - P_{-G_4(l)}\right)\left[\frac{\mathcal{E}_{-G_4(l)} + x_9}{\beta}\right]$$

$$\Delta_4^{A9}(l) \quad := \quad \Gamma^l_{G_4} \qquad \textbf{(Budget Balancing term)}$$

$$\mathcal{E}_{-G_4(l)} := x_{G_1}(l) + x_{G_2}(l) + x_{G_3}(l) - c_l$$

$$\eta^l_+ := \max\{0, \frac{x_{G_1}(l) + x_{G_2}(l) + x_{G_3}(l) + x_{G_4}(l) - c_l}{\gamma}\}$$

$$P_{-G_4}(l) := \frac{1}{3}\left(P_{G_1}(l) + P_{G_2}(l) + P_{G_3}(l)\right),$$

Outcome function: f

If $\mathbf{A_8}$ requests the **maximum** bandwidth in $\mathbf{G_4}$ at link l, and A_9 does not request the maximum bandwidth at l:

$$t^l_{A_9} = \pi_8 \left(\mathcal{E}_{-G_4(l)} + x_{G_4}(l)\right)\left(1 - \Delta_1^{A9}(l)\right).$$

Properties of the Mechanism

Properties of the Mechanism

[Existence and Feasibility]

- There exists at least one **pure** NE for the game induced by the game form.
- If $\mathbf{m}^*$ is a *NE* point of the game induced by the game form, then the allocation $\mathbf{x}^*$ is a feasible solution of Problem $(\mathbf{P_C})$.

‘

Properties of the Mechanism

[Properties of NE]

Let m^* be a *NE* of the game induced by game form. Then for every $l \in \mathbf{L}$ we have,

- **Each group** using the link bids the the SAME price $= P^*_{G(l)}$
- $P^*_{G(l)} \left[\frac{c_l - \textbf{Sum} \text{ of the } \textbf{demands} \text{ requested by the groups}}{\beta} \right] = 0.$

‘

Properties of the Mechanism

[**Budget Balance**]

The proposed mechanism/game form is always balanced budget at every allocation corresponding to NE messages.

‘

Properties of the Mechanism

[Individually Rationality]

The specified mechanism is **individually rational at all NE**, i.e., at every NE, the corresponding allocation is weakly preferred by all agents to the initial allocation.

$$\mathbf{U}_{(j,G_i)}(x^*_{(j,G_i)}) - t_{(j,G_i)}(\mathbf{m}^*) \geqslant 0, \qquad \forall\ (j, G_i)$$

‘

Properties of the Mechanism

[Nash Implementation]

Consider the allocation $(f(\mathbf{m}^*) = (\mathbf{x}^*, \mathbf{t}^*))$ corresponding to **any NE** message $\mathbf{m}^*$.
Then $\mathbf{x}^*$ is an **optimal solution** of the
centralized problem ($\mathbf{P_C}$).

Problem (P_C)

$$\max_{(\boldsymbol{x}_{\mathcal{N}})} \quad \sum_{G_i \in \mathcal{N}} \sum_{(j,G_i) \in G_i} U_{(j,G_i)}(x_{(j,G_i)})$$

$$\text{s.t.} \quad \sum_{G_i \in Q_l} \max_{(j,G_i) \in G_i(l)} x_{(j,G_i)} \leqslant c_l \qquad \forall\, l \in \mathbf{L}$$

$$x_{(j,G_i)} \geqslant 0 \qquad \forall\, (j, G_i)$$

Conclusion

- Addressed the Multi-rate Multicast service provisioning problem for the **first** time with **strategic** agents.
- Designed a game form that
  (1) is **individually rational**.
  (2) results in **budget balance**.
  (3) **All** the NE of the game induced by the game form are **efficient**.
- The proposed game form is the **first** game form that captures the features of *a Market problem* and *a Public good problem*, simultaneously.

Open Problems

- Tatonement processes for determination of NE.

- Dynamic resource allocation problems

References

- A. Kakhbod and D. Teneketzis, “An Efficient Game Form for Multi-rate Multicast Service Provisioning”, *IEEE Journal on Selected Areas in Communication, Special Issue on the Economics of Communication Networks and Systems, Vol. 30, No. 10, December 2012, pp 2093-2104.*

- Correction to “An Efficient Game Form for Multi-rate Multicast Service Provisioning”, *IEEE Journal on Selected Areas in Communication, Vol. 30, No. 7, July 2013.*